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A computation of fin-line impedence

Kim, Byungyong

Monterey, California. Naval Postgraduate School

NAVAL POSTGRADUATE SCHOOL

Monterey, California

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A COMPUTATION OF FIN-LINE
IMPEDANCE

by

Byungyong Kim

December 1984

Thesis Advisor: J. B. Knorr

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The spectral domain solution for the wavelength and characteristic impedance of a millimeter wave fin-line was originally published by Knorr and Shayda. The dispersion equations were subsequently reformulated by Knorr in a form more suitable for numerical computation.

This thesis presents a reformulation of the equations for characteristic impedance for the same purpose. The equations

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A Computation of Fin-line Impedance

by

Kim, Byungyong
Major, Korean Air Force
B.S.E.E., Korean Air Force Academy, 1977

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The spectral domain solution for the wavelength and characteristic impedance of a millimeter wave fin-line was originally published by Knorr and Shayda. The dispersion equations were subsequently reformulated by Knorr in a form more suitable for numerical computation.

This thesis presents a reformulation of the equations for characteristic impedance for the same purpose. The equations have been used to implement a computer program, FINIMP. The program runs smoothly without the overflow and underflow problems experienced by Knorr and Shayda. FINIMP data is compared with other existing data and good agreement is shown to establish the correctness of the FINIMP numerical results.

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I. INTRODUCTION

A. BACKGROUND AND RELATED WORK

The study of electromagnetic energy transmission is but one important area in microwave and millimeter-wave engineering, where the electromagnetic waves are travelling through some transmission medium, which provides the link between the transmitting and receiving part of a transmission system.

In recent years, fin-line has gained in importance as a transmission medium in millimeter wave circuit constructions [Ref. 1 - 4]. Fin-line has been found superior to microstrip at millimeter wavelengths as the former provides eased production tolerances, low dispersion, broad single mode bandwidth, moderate attenuation, better compatibility with hybrid devices, greater freedom from radiation and higher mode propagation, combined with the ability to construct simple transitions to conventional rectangular waveguide.

Figure 1.1 shows a 3-dimensional view of fin-line. The stucture may be viewed as a slotline with a shield, a ridged waveguide with dielectric or a slab loaded waveguide with fins.

The fin-line structure was first proposed for millimeter wave integrated circuits in 1974 by Meier [Ref. 1]. An early paper by Meier described the propagation mode as a variation of dominant mode in ridged waveguide. His procedure requires a test measurement to determine the equivalent dielectric constant of the fin-line structure. This is both expensive and time consuming. Knorr and Kuchler [Ref. 5] presented a frequency dependent hybrid-mode analysis of slot line with open boundary using the spectral domain technique

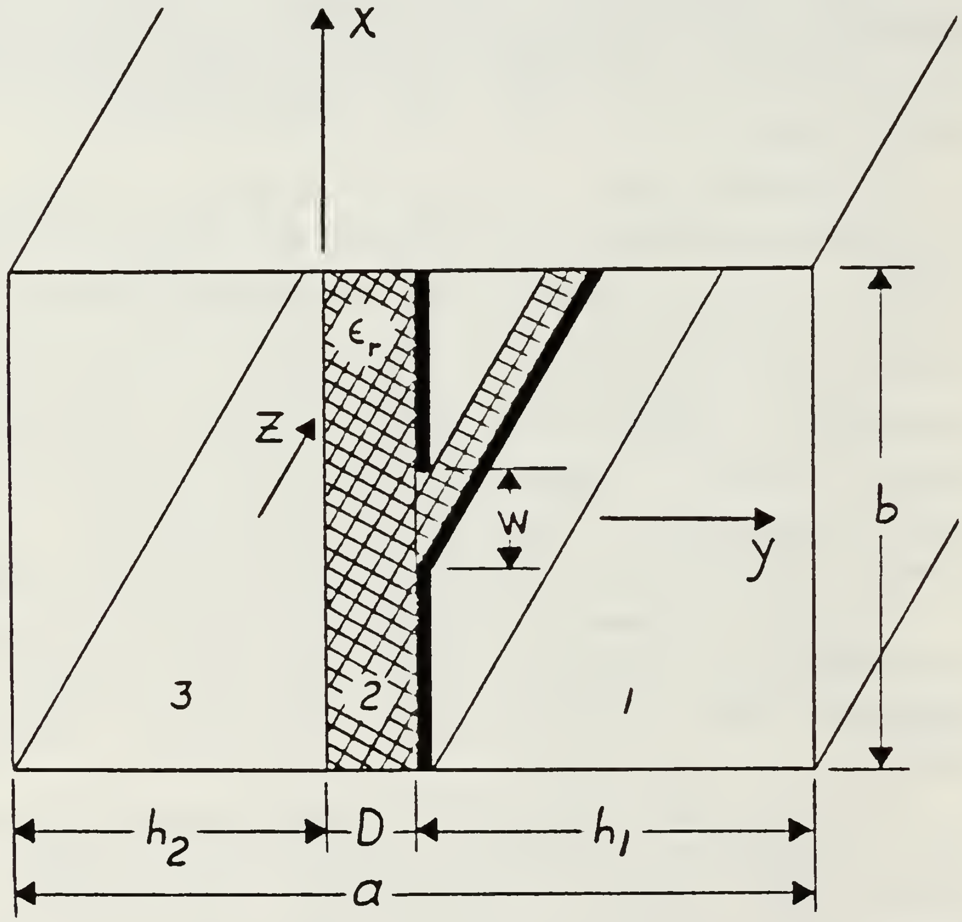

Figure 1.1 3-Dimensional View of Fin-line Structure.

which was suggested by Itoh and Mittra [Ref. 6], in 1975. Subsequently, the paper of prime importance in the establishment of spectral domain technique for analyzing the fin-line structure, so-called shielded slotline, was pesented by Knorr and Shayda in 1980 [Ref. 3]. One of the advantages of the spectral domain approach is that it is numerically more efficient than the conventional methods that work directly in the space domain. This is due primarily to the fact that the process of Fourier transformation of the coupled integral equations in the space domain yields a pair of algebraic equations in the spectral domain that are relatively easier to handle. Another important advantage is that the Green's function takes a much simpler form in the transform domain, as compared to the space domain where no convenient form of the Green's function is known to exist.

B. PURPOSE

In solving the open boundary slotline problem, only exponential functions arise and there are no numerical problems during computation. For a closed boundary structure, however, hyperbolic functions are required. Knorr and Shayda [Ref. 3] found that overflow and underflow problems resulted during numerical computations. To eliminate these problems, extensive algebraic manipulation of the spectral domain equations is required.

This thesis presents the method to calculate the characteristic impedance of fin-line without overflow and underflow in equations. Therefore, this thesis is a direct extention of Knorr's work. This work is accomplished by lots of algebraic manipulations. In particular, since hyperbolic sine and cosine functions cause overflow and underflow errors, the equations developed by Knorr need to be put in a form where only the hyperbolic tangent function appears. The characteristic impedance is computed after the spectral domain technique to find the dispersion characteristic.

Following the theoretical analysis, an explanation of the computer program used in determination of characteristic impedance is presented. Numerical results are then compared with known data for ridged waveguide, slab loaded waveguide, slot line and fin-line [Ref. 3]. This comparison establishes the accuracy of the numerical results.

II. THEORETICAL ANALYSIS OF FIN LINE

A. FIELD AND BOUNDARY CONDITIONS

The fin-line supports a hybrid field. The axial components of TM and TE modes are then

$$E_z = K_c^2 \phi^e(x,y)\, e^{\Gamma z} \qquad \text{(eqn 2.1)}$$

$$H_z = K_c^2 \phi^h(x,y)\, e^{\Gamma z} \qquad \text{(eqn 2.2)}$$

where the scalar potential functions ϕ^e, ϕ^h satisfy the Helmholz equation, and we assume lossless propagation so that $\Gamma = \pm j\beta$.

Further

$$K_{ci}^2 = K_i^2 - \beta^2 \qquad \text{(eqn 2.3)}$$

with $k_i^2 = \omega^2 \mu_i \varepsilon_i$, $i = 1, 2, 3,$ defining spatial region of finline as stated in Figure 1.1.

Through Maxwell's curl equations the transverse field components are then determined by these axial components and can be given as

$$E_x = \left(\Gamma \frac{\partial \phi^e}{\partial x} - j\omega\mu \frac{\partial \phi^h}{\partial y}\right) e^{\Gamma z} \qquad \text{(eqn 2.4)}$$

$$E_y = \left(\Gamma \frac{\partial \phi^e}{\partial y} + j\omega\mu \frac{\partial \phi^h}{\partial x}\right) e^{\Gamma z} \qquad \text{(eqn 2.5)}$$

$$H_x = \left(\Gamma \frac{\partial \phi^h}{\partial x} + j\omega\varepsilon \frac{\partial \phi^e}{\partial y}\right) e^{\Gamma z} \qquad \text{(eqn 2.6)}$$

$$H_y = \left(\Gamma \frac{\partial \phi^h}{\partial y} - j\omega\varepsilon \frac{\partial \phi^e}{\partial x} \right) e^{\Gamma z} \qquad \text{(eqn 2.7)}$$

where propagation in the z direction is assumed. We will also assume here that $\varepsilon_1 = \varepsilon_3 = \varepsilon_0$ and $\varepsilon_2 = \varepsilon_0 \varepsilon_r$.

At $y = h_1 + D$:

Applying boundary conditions at the walls in region 1, tangential field components must be zero.

$$E_{z1}(x, h_1 + D, z) = 0 \qquad \text{(eqn 2.8)}$$

$$E_{x1}(x, h_1 + D, z) = 0 \qquad \text{(eqn 2.9)}$$

At $y = D$:

At the interface between region 1 and region 2, tangential field components must be continuous.

$$E_{z1}(x, D, z) = E_{z2}(x, D, z) \qquad \text{(eqn 2.10)}$$

$$E_{x1}(x, D, z) = E_{x2}(x, D, z) \qquad \text{(eqn 2.11)}$$

Also the electric fields at $y = D$ will exist only in the slot.

$$E_{z1}(x, D, z) = \begin{cases} 0 & |x| \geq \frac{W}{2} \\ e_z(x)\, e^{\Gamma z} & |x| < \frac{W}{2} \end{cases} \qquad \text{(eqn 2.12)}$$

$$E_{x1}(x, D, z) = \begin{cases} 0 & |x| \geq \frac{W}{2} \\ e_x(x)\, e^{\Gamma z} & |x| < \frac{W}{2} \end{cases} \qquad \text{(eqn 2.13)}$$

Similarly, tangential magnetic fields must be discontinuous by corresponding surface current densities.

$$H_{z1}(x, D, z) - H_{z2}(x, D, z) = \begin{cases} j_x(x)\, e^{\Gamma z} & |x| \geq \frac{w}{2} \\ 0 & |x| < \frac{w}{2} \end{cases}$$

(eqn 2.14)

$$H_{x1}(x, D, z) - H_{x2}(x, D, z) = \begin{cases} j_z(x)\, e^{\Gamma z} & |x| \geq \frac{w}{2} \\ 0 & |x| < \frac{w}{2} \end{cases}$$

(eqn 2.15)

AT y = 0 :

The tangential field components at the interface between region 2 and 3 must also be continuous.

$$E_{z2}(x, 0, z) = E_{z3}(x, 0, z) \qquad \text{(eqn 2.16)}$$

$$E_{x2}(x, 0, z) = E_{x3}(x, 0, z) \qquad \text{(eqn 2.17)}$$

$$H_{z2}(x, 0, z) = H_{z3}(x, 0, z) \qquad \text{(eqn 2.18)}$$

$$H_{x2}(x, 0, z) = H_{x3}(x, 0, z) \qquad \text{(eqn 2.19)}$$

At $y = -h_2$:

Once again at the shield wall in region 3, the tangential field components must be zero.

$$E_{z3}(x, -h_2, z) = 0 \qquad \text{(eqn 2.20)}$$

$$E_{x3}(x, -h_2, z) = 0 \qquad \text{(eqn 2.21)}$$

At $x = \pm b/2$:

The final boundary conditions occur at $x = b/2$ where the tangential components must be zero in all regions.

$$E_{zi}(\pm b/2, y, z) = 0 \qquad \text{(eqn 2.22)}$$

$$E_{xi}(\pm b/2, y, z) = 0 \qquad \text{(eqn 2.23)}$$

B. SPECTRAL DOMAIN APPROACH TO DISPERSION CHARACTERISTC

The scalar potential functions can be transformed into the spectral domain via Fourier transform defined as

$$F\{\phi(x,y)\} = \tilde{\Phi}(\alpha_n, y) = \int_{-\infty}^{\infty} \phi(x,y)\, e^{j\alpha_n x}\, dx . \qquad \text{(eqn 2.24)}$$

The scalar potential functions satisfy Helmholz's equations in the three spatial regions, thus

$$\nabla_t^2 \phi_i + K_{ci}^2 \phi_i = 0 \qquad \text{(eqn 2.25)}$$

where denotes the two dimensional Laplacian operator in the transverse (x,y) direction. The Helmholtz equation 2.25 are transformed into

$$\frac{\partial^2 \tilde{\Phi}_i(\alpha_n, y)}{\partial y^2} = (\alpha_n^2 - K_{ci}^2)\, \tilde{\Phi}_i(\alpha_n, y) \qquad \text{(eqn 2.26)}$$

where $\gamma_i^2 = \alpha_n^2 - K_{ci}^2 = \alpha_n^2 + \beta^2 - K$. Above equation has solutions after applying boundary conditions at $y = D + h_1$, and $y = -h_2$.

$$\tilde{\Phi}_1^e(\alpha_n, y) = A^e(\alpha_n) \sinh \gamma_1 (D + h_1 - y) \qquad \text{(eqn 2.27)}$$

$$\Phi_2^e(\alpha_m, y) = B^e(\alpha_m)\sinh \gamma_2 y + C^e(\alpha_m)\cosh \gamma_2 y \qquad \text{(eqn 2.28)}$$

$$\Phi_3^e(\alpha_m, y) = D^e(\alpha_m)\sinh \gamma_3 (h_2 + y) \qquad \text{(eqn 2.29)}$$

$$\Phi_1^h(\alpha_m, y) = A^h(\alpha_m)\cosh \gamma_1 (D + h_1 - y) \qquad \text{(eqn 2.30)}$$

$$\Phi_2^h(\alpha_m, y) = B^h(\alpha_m)\sinh \gamma_2 y + c^h(\alpha_m)\cosh \gamma_2 y \qquad \text{(eqn 2.31)}$$

$$\Phi_3^h(\alpha_m, y) = D^h(\alpha_m)\cosh \gamma_3 (h_2 + y) \qquad \text{(eqn 2.32)}$$

where

$$\alpha_m = \begin{cases} \dfrac{m2\pi}{b} & \phi^h \text{ even} \qquad \text{(eqn 2.33)} \\[2ex] \dfrac{(2m-1)\pi}{b} & \phi^h \text{ odd} \qquad \text{(eqn 2.34)} \end{cases}$$

The following observation about the solutions in region 2 must be emphasized. Any wave on this inhomogeneous waveguide structure will partly travel through air and partly through the dielectric slab. It is important to observe at this point that γ_i^2 may be less than zero in any of the three regions of the structure under certain conditions. When $\alpha_m = 0$ and k_i approaches k_o (where k_o is the wave number for free space), β is less than k_i and so $\gamma_i^2 < 0$. Under this condition the hyperbolic functions in all three regions are replaced by trigonometric functions. If $k_o < \beta < k_2$, then γ_1^2 and γ_3^2 are greater than zero and $\gamma_2^2 < 0$ for some values of and the trigonometric functions replace the hyperbolic functions in the spatial region 2 only. This suggests that the nature of the field is dependent upon the values of the transform variable, . For the conditions when $\gamma_i^2 < 0$, γ_i'' replaces γ_i such that $(\gamma_i'')^2 = -\gamma_i^2$. The eight unknown coefficients A through D are not independent, but can be related to each other through the continuity conditions of the field components at the interfaces between the three spatial regions.

If we denote the Fourier transforms of x- and z-directed current density and electric field component by

$$\tilde{E}_x(\alpha_m) = F\{e_x(x)\} \qquad \text{(eqn 2.35)}$$

$$\tilde{E}_z(\alpha_m) = F\{e_z(x)\} \qquad \text{(eqn 2.36)}$$

$$\tilde{J}_x(\alpha_m) = F\{j_x(x)\} \qquad \text{(eqn 2.37)}$$

$$\tilde{J}_z(\alpha_m) = F\{j_z(x)\} . \qquad \text{(eqn 2.38)}$$

The resulting set of linear equations may be written in matrix form as follows:

$$[M_E] \begin{bmatrix} A^e \\ B^e \\ \cdot \\ \cdot \\ \cdot \\ C^h \\ D^h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{E}_x \\ \tilde{E}_z \end{bmatrix} \qquad \text{(eqn 2.39)}$$

$$[M_J] \begin{bmatrix} A^e \\ B^e \\ \cdot \\ \cdot \\ \cdot \\ C^h \\ D^h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{J}_x \\ \tilde{J}_z \end{bmatrix} . \qquad \text{(eqn 2.40)}$$

The matrices [ME] and [MJ] differ in only the last two rows. Each is a square 8 x 8 matrix. Using equations (2.39) and (2.40), we may write

$$[M_J][M_E]^{-1}\begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{E}_x \\ \tilde{E}_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{J}_x \\ \tilde{J}_z \end{bmatrix} .$$

(eqn 2.41)

From equation (2.41), using the four elements in the lower right hand corner of the matrix MJ ME , we obtain

$$\begin{bmatrix} \tilde{G}_1(\alpha_n,\beta) & \tilde{G}_2(\alpha_n,\beta) \\ \tilde{G}_3(\alpha_n,\beta) & \tilde{G}_4(\alpha_n,\beta) \end{bmatrix}\begin{bmatrix} \tilde{E}_x(\alpha_n) \\ \tilde{E}_z(\alpha_n) \end{bmatrix} = \begin{bmatrix} \tilde{J}_x(\alpha_n) \\ \tilde{J}_z(\alpha_n) \end{bmatrix} . \quad \text{(eqn 2.42)}$$

where the 2 x 2 matrix $[\tilde{G}]$ contains the Fourier transforms of the components of the dyadic Green's function for this structure.

A solution to equation (2.42) is obtained using the Method of Moments [Ref. 7]. For this problem, we have chosen to approximate the field between the fins as shown in Figure 2.1:

$$e_x(x) = \begin{cases} 1 & |x| \le \frac{W}{2} \\ 0 & \text{elsewhere} \end{cases} \quad \text{(eqn 2.43)}$$

$$e_z(x) = 0 \ . \quad \text{(eqn 2.44)}$$

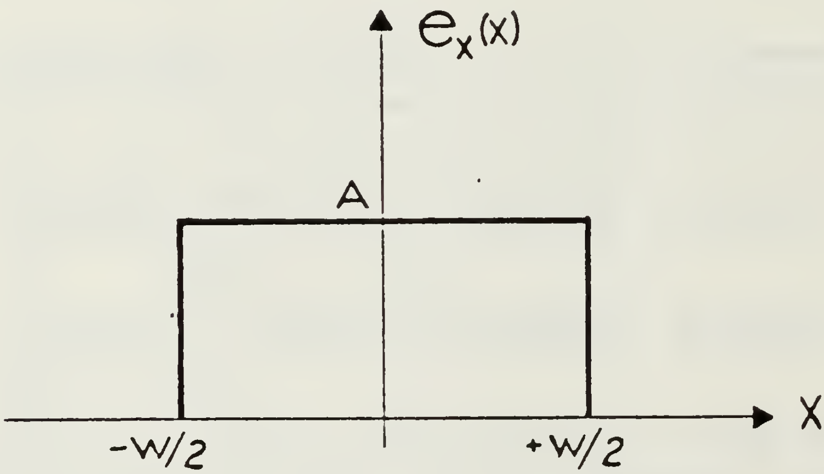

Figure 2.1 Assumed Electric Field Component in Slot in x-direction versus x for Fin-line.

The dispersion problem is now reduced to the form

$$\sum_{n=-\infty}^{\infty} \tilde{G}_1(\alpha_n, \beta) \left| \tilde{E}_x(\alpha_n) \right|^2 = 0 \qquad \text{(eqn 2.45)}$$

where

$$\tilde{E}_x(\alpha_n) = \int_{-\frac{W}{2}}^{\frac{W}{2}} e_x(x)\, e^{j\alpha_n x}\, dx = AW \frac{\sin(\alpha_n W/2)}{\alpha_n W/2} \quad .$$

A numerical search for the value of which satisfies equation (2.45) yields the propagation constant for the dominant fin-line mode.

From the equation (2.39)

$$m_{11} A^e + m_{12} B^e + m_{13} C^e = 0 \qquad \text{(eqn 2.46)}$$

$$m_{21} A^e + m_{22} B^e + m_{23} C^e +$$
$$m_{25} A^h + m_{26} B^h + m_{27} C^h = 0 \qquad \text{(eqn 2.47)}$$

$$m_{81}^{\varepsilon} A^e = D^2 \tilde{E}_z \qquad \text{(eqn 2.48)}$$

$$m_{71}^{\varepsilon} A^e + m_{75}^{\varepsilon} A^h = D^2 \tilde{E}_x \qquad \text{(eqn 2.49)}$$

$$m_{33} C^e + m_{34} D^e = 0 \qquad \text{(eqn 2.50)}$$

$$m_{43} C^e + m_{44} D^e + m_{46} B^h + m_{48} D^h = 0 \qquad \text{(eqn 2.51)}$$

$$m_{57} C^h + m_{58} D^h = 0 \qquad \text{(eqn 2.52)}$$

$$m_{62} B^e + m_{64} D^e + m_{67} C^h + m_{68} D^h = 0 \qquad \text{(eqn 2.53)}$$

From equations (2.50) and (2.52)

$$D^e = -\frac{m_{33}}{m_{34}} C^e \qquad \text{(eqn 2.54)}$$

$$D^h = -\frac{m_{57}}{m_{58}} C^h . \qquad \text{(eqn 2.55)}$$

Using equations (2.48) and (2.49)

$$A^e = \left(\frac{1}{m_{81}^{\varepsilon}}\right) D^2 \tilde{E}_z \qquad \text{(eqn 2.56)}$$

$$A^h = \left(\frac{1}{m_{75}^{\varepsilon}}\right) D^2 \tilde{E}_x - \left(\frac{m_{71}^{\varepsilon}}{m_{75}^{\varepsilon} m_{81}^{\varepsilon}}\right) D^2 \tilde{E}_z . \qquad \text{(eqn 2.57)}$$

Substituting in equations (2.51) and (2.53), we obtain

$$B^e = \left(\frac{m_{64} m_{33}}{m_{62} m_{34}}\right) C^e + \left(\frac{m_{68} m_{57}}{m_{62} m_{58}} - \frac{m_{67}}{m_{62}}\right) C^h \qquad \text{(eqn 2.58)}$$

$$B^h = \left(\frac{m_{44} m_{33}}{m_{46} m_{34}} - \frac{m_{43}}{m_{46}}\right) C^e + \left(\frac{m_{48} m_{57}}{m_{46} m_{58}}\right) C^h . \qquad \text{(eqn 2.59)}$$

Substituting equations (2.48), (2.49), (2.51), and (2.53) in equations (2.46) and (2.47) we obtain

$$C^e = \left(\frac{a_{12}}{|D|}\right)\left(\frac{m_{25}}{m_{75}^{\varepsilon}}\right) D^2 \tilde{E}_x - \left(\frac{a_{22}}{|D|}\right)\left(\frac{m_{11}}{m_{81}^{\varepsilon}}\right) D^2 \tilde{E}_z$$
$$- \left(\frac{a_{12}}{|D|}\right)\left(\frac{m_{25} m_{71}^{\varepsilon}}{m_{75}^{\varepsilon} m_{81}^{\varepsilon}} - \frac{m_{21}}{m_{81}^{\varepsilon}}\right) D^2 \tilde{E}_z \qquad \text{(eqn 2.60)}$$

$$C^h = -\left(\frac{a_{11}}{|D|}\right)\left(\frac{m_{25}}{m_{75}^{\varepsilon}}\right) D^2 \tilde{E}_x + \left(\frac{a_{11}}{|D|}\right)\left(\frac{m_{25} m_{71}^{\varepsilon}}{m_{75}^{\varepsilon} m_{81}^{\varepsilon}} -\right.$$
$$\left.\frac{m_{21}}{m_{81}^{\varepsilon}}\right) D^2 \tilde{E}_z + \left(\frac{a_{21}}{|D|}\right)\left(\frac{m_{11}}{m_{81}^{\varepsilon}}\right) D^2 \tilde{E}_z \qquad \text{(eqn 2.61)}$$

where

$$|D| = a_{11}a_{22} - a_{21}a_{12}$$

$$= j\,(d_{11}d_{22} - d_{21}d_{12})(\gamma_2 D)\sinh(\gamma_2 D)\cosh(\gamma_2 D)\,.$$

We define

$$a_{11} = \left[m_{13} + m_{12}\left(\frac{m_{64}\,m_{33}}{m_{62}\,m_{34}}\right)\right]$$

$$= d_{11}\cosh(\gamma_2 D) \qquad \text{(eqn 2.62)}$$

$$a_{12} = \left[m_{12}\left(\frac{m_{68}\,m_{57}}{m_{62}\,m_{58}}\right) - \frac{m_{67}}{m_{62}}\right]$$

$$= j\,d_{12}(\gamma_2 D)\sinh(\gamma_2 D) \qquad \text{(eqn 2.63)}$$

$$a_{21} = \left[m_{23} + m_{22}\left(\frac{m_{64}\,m_{33}}{m_{62}\,m_{34}}\right) + m_{26}\left(\frac{m_{44}\,m_{33}}{m_{46}\,m_{34}} - \frac{m_{43}}{m_{46}}\right)\right]$$

$$= d_{21}\cosh(\gamma_2 D) \qquad \text{(eqn 2.64)}$$

$$a_{22} = \left[m_{27} + m_{26}\left(\frac{m_{48}\,m_{57}}{m_{46}\,m_{58}}\right) + m_{22}\left(\frac{m_{68}\,m_{57}}{m_{62}\,m_{58}} - \frac{m_{67}}{m_{62}}\right)\right]$$

$$= j\,d_{22}(\gamma_2 D)\sinh(\gamma_2 D) \qquad \text{(eqn 2.65)}$$

Also we define

$$d_{11} = -(K_{c2}D)^2\left[1 + \frac{(\omega\varepsilon_3 D)(\gamma_3 D)^2 (K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2} \cdot \frac{(\gamma_2 D)\tanh(\gamma_2 D)}{(\gamma_3 D)\tanh[(\gamma_3 D)(h_2/D)]}\right] \qquad \text{(eqn 2.66)}$$

$$d_{12} = (K_{c2}D)^2\left[\frac{(\alpha_m D)(\beta D)}{(\omega\varepsilon_2 D)(\gamma_2 D)}\left(\frac{(K_{c2}D)^2}{(K_{c3}D)^2} - 1\right)\right] \qquad \text{(eqn 2.67)}$$

$$d_{21} = -(\alpha_m D)(\beta D)\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2} \cdot \frac{(\gamma_2 D)\tanh(\gamma_2 D)}{(\gamma_3 D)\tanh[(\gamma_3 D)(h_2/D)]} + \frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right] \qquad \text{(eqn 2.68)}$$

$$d_{22} = (\omega\mu D)\left[\left(1 + \frac{(K_{c2}D)^2(\gamma_3 D)\tanh[(\gamma_3 D)(h_2/D)]}{(K_{c3}D)^2(\gamma_2 D)\tanh(\gamma_2 D)}\right) + \left[\frac{(\alpha_m D)^2(\beta D)^2}{(\omega\mu D)(\omega\varepsilon_2 D)(\gamma_2 D)^2}\right]\left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2} - 1\right]\right]. \qquad \text{(eqn 2.69)}$$

Nomalized constants are defined as follows

$$(K_{c1}D)^2 = (K_{c3}D)^2 = (2\pi)^2\left[1 - (\lambda/\lambda')^2\right](D/\lambda)^2$$

$$(K_{c2}D)^2 = (2\pi)^2\left[\varepsilon_r - (\lambda/\lambda')^2\right](D/\lambda)^2$$

$$\omega\mu D = 240\,\pi^2\,(D/\lambda)$$

$$\omega\varepsilon_1 D = \omega\varepsilon_3 D = \frac{1}{60}(D/\lambda)$$

$$\omega \varepsilon_2 D = \varepsilon_r / 60 \, (D/\lambda)$$

$$\beta D = 2\pi \, (D/\lambda)(\lambda/\lambda')$$

$$\alpha_m D = \begin{cases} m \, 2\pi \left(\frac{D}{b}\right) & \phi^h \text{ even} \\ (2m-1)\pi \left(\frac{D}{b}\right) & \phi^h \text{ odd} \end{cases}$$

$$(\gamma_1 D)^2 = (\gamma_3 D)^2 = (\alpha_m D)^2 + (2\pi)^2 [(\lambda/\lambda')^2 - 1] \, (D/\lambda)^2$$

$$(\gamma_2 D)^2 = (\alpha_m D)^2 + (2\pi)^2 [(\lambda/\lambda')^2 - \varepsilon_r] \, (D/\lambda)^2 .$$

C. CHARACTERISTIC IMPEDANCE

The definition of the characteristic impedance for an ideal TEM transmission line is uniquely given by static quantities. Since the fin-line supports a hybrid mode, no unique definition of the characteristic impedance can be found.

A useful definition, however, is

$$Z_0 = \frac{V_0^2}{2P_{avg}} \qquad \text{(eqn 2.70)}$$

where V_0 is the slot voltage defined as

$$V_0 = \int_{-W/2}^{W/2} A \, dx = 1 \quad . \qquad \text{(eqn 2.71)}$$

$e_x(x)$ is arbitrarily selected as $1/W$ so that $W^* e_x(x) = 1$. P_{avg} is given by

$$P_{avg} = \frac{1}{2} \mathrm{Re} \iint_S \bar{E} \times \bar{H}^* \cdot \hat{a}_z \, da$$

$$= \frac{1}{2} \mathrm{Re} \iint_S (E_x H_y^* - E_y H_x^*) \, dx dy \; . \qquad \text{(eqn 2.72)}$$

Pasval's theorem is applied to eq(2.88) to obtain

$$P_{avg} = \frac{1}{2} \mathrm{Re} \frac{1}{b} \sum_{m=-\infty}^{\infty} \int_{-h_2}^{D+h_1} [E_x(\alpha_m, y) H_y^*(\alpha_m, y) - E_y(\alpha_m, y) H_x^*(\alpha_m, y)] \, dy$$

This expression must be evaluated in each of the three regions of the fin-line shown in Figure 1.1 Therefore the power flow may be expressed by

$$P_{avg} = \frac{1}{2b} \sum_{m=-\infty}^{\infty} (P_1 + P_2 + P_3) \qquad \text{(eqn 2.73)}$$

Since P_{avg} can be determined after finding the value of λ'/λ. Equations including only hyperbolic tangent functions and the slot voltage V_o have been developed the characteristic impedance. The lengthy algebraic manipulations are shown in appendix B.

III. COMPUTER PROGRAMMING

A. NUMERICAL ANALYSIS

The computation of characteristic impedance is based upon the solution to the dispersion chracteristic problem for the fin-line under consideration. In other words, the wave propagation constant or wavelength ratio λ'/λ must be known before any other investigations can be started since only in this case are the scalar potential functions in the transform domain known. The computer program for the wavelength ratio λ'/λ is already developed by Prof. Knorr. The next task is the preparation of the appropriate equations for the numerical evaluation of the time average power flow. For ease in numerical calculations and for programming purposes all geometric parameters are normalized as follows;

h_1/D ; fin location relative to the positive "y" side wall normalized with respect to D

h_2/D ; fin location relative to the negative "y" side wall normalized with respect to D

b/D ; waveguide height normalized with respect to D.

It is observed that for the power flow computations the infinite integration in equation (2.88) is replaced by an infinite series due to the discrete nature of the transform variable α_n. Preliminary numerical investigations of the coefficients of these series indicated an even distribution with respect to the variable so that computation time can be saved by summing over a half interval only.

The time average power flow equations are prepared for six different cases as follows;

case 1 ; $(\gamma_1 D)^2 < 0$ in region 1

case 2 ; $(\gamma_1 D)^2 > 0$ in region 1

case 3 ; $(\gamma_2 D)^2 < 0$ in region 2 and $(\gamma_3 D)^2 < 0$

in region 3

case 4 ; $(\gamma_2 D)^2 < 0$ in region 2 and $(\gamma_3 D)^2 > 0$ in region 3

case 5 ; $(\gamma_2 D)^2 > 0$ in region 2 and $(\gamma_3 D)^2 < 0$ in region 3

case 6 ; $(\gamma_2 D)^2 > 0$ in region 2 and $(\gamma_3 D)^2 > 0$ in region 3

After this preparation of the equations for the computer programming, the only remaining task is to investigate the numerical integration with regard to its limits and the procedure to be used. One of the limits of the summation discussed previously extends to infinity so it is necessary to determine an appropriate at which to truncate the computations.

From the representative examples shown in Figure 3.1, it is seen that the coefficients in this infinite series are found to decay rapidly so that a finite approximation yields good results. Figure 3.1 shows the characteristic impedance as a function of n, the number of terms in the truncated series, for various slot widths. The value $n = 50$ is sufficiently large to be considered infinite for all practical purposes. It is noted that when $w/b = 1$, $Ex(\alpha_n) = 0$ for $n > 0$. Therefore, whenever $w/b = 1$ only the $n = 0$ term is computed in impedance calculations.

B. COMPUTER PROGRAMMING AND LIMITATION

The first part of the FINIMP computer program finds the λ'/λ which makes equation (2.45) equal to zero. The assumption is that λ'/λ is between 0.1 and 3 in the giga hertz range and continuously decreases as the frequecy is increased. Since this program is very sensitive there is a possibility that the wrong value is searched. In that case, we can easily notice because there is abruptly jumped value.

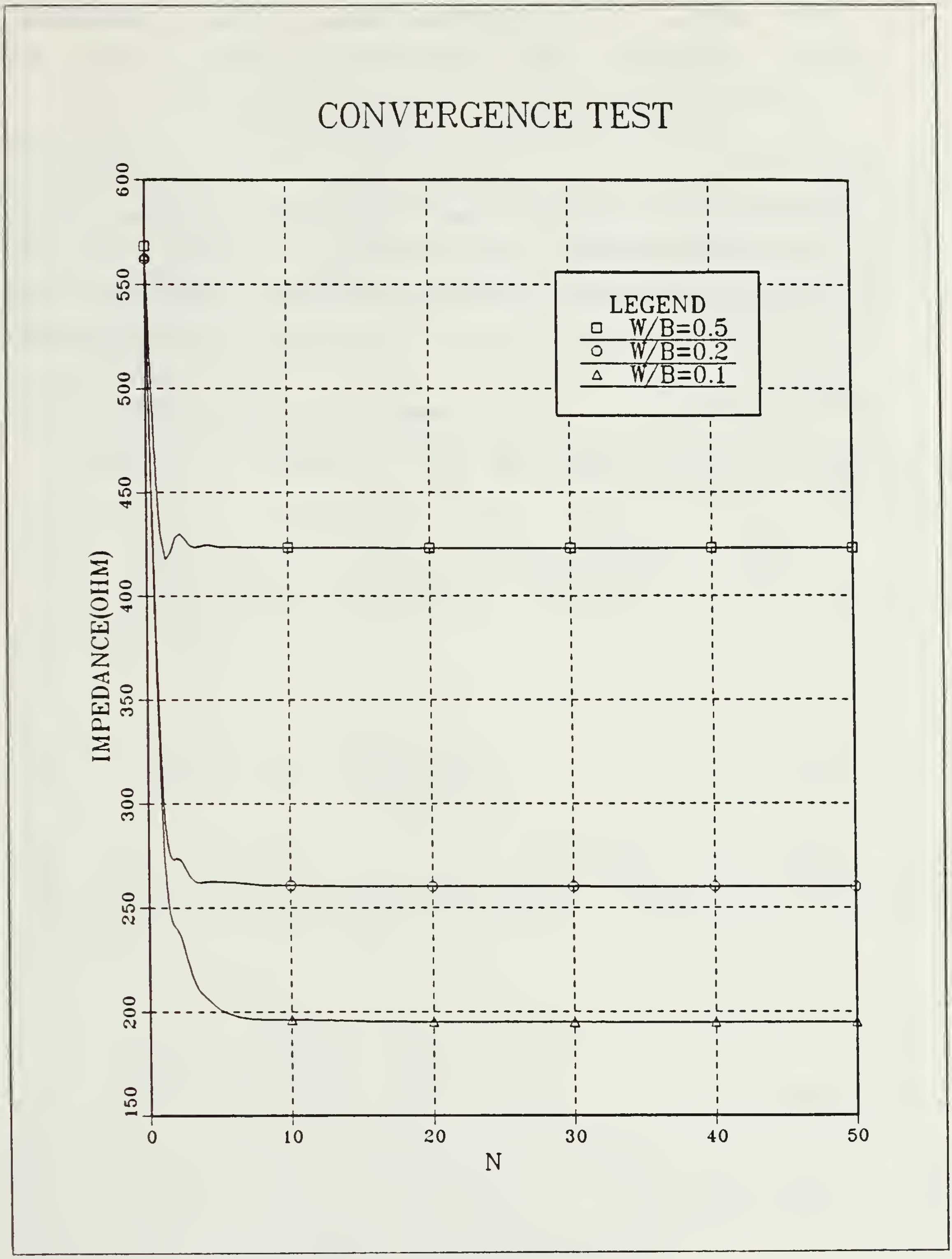

Figure 3.1 Characteristic Impedance Z vs. Iteration for a Fin-line with b/D=18.8 h_1/D=18.8 h_2/D=17.8 D=.005" ε_r=2.2 f=40.0GHZ..

At that time, if the assumed range of λ'/λ is resticted to a smaller interval than before the correct value can be obtained.

The rest of the FINIMP computer program is the computation of characteristic impedance. Since all equations are composed of hyperbolic tangent functions, there is no overflow in the computer program. But if $(\gamma_{\lambda} D)^2 < 0$ tangent functions replace the hyperbolic tangent functions. At that time, there is a possibity of overflow because tan $n\pi/2$ is infinite for n = 1, 3, 5, ... In that case the tangent function should be set to some value which is the maximum value of computer ability $(46^{63} \cdot (1-16^{-6}) > |\tan(x)|)$.

IV. NUMERICAL RESULTS AND COMPARISONS

To check the accuracy of the numerical results gererated by the computer program, comparisons are made with data available in the literature for ridged waveguide, slab loaded waveguide, slotline, empty waveguide and fin-line as outlined in [Ref. 3].

A. RIDGED WAVEGUIDE

When = 1 and w/D < 1 or when the dielectric substrate thickness D is reduced to zero, the fin-line structure becomes ridged waveguide with zero thickness ridges.

The impedance of the ridged waveguide has the following relations [Ref. 8],

$$Z_0 = \frac{Z_{0\infty}}{\left[1 - (\lambda/\lambda_c)^2\right]^{1/2}} \quad . \qquad \text{(eqn 4.1)}$$

When the guide width and the slot width are equal, the ridged waveguide becomes an ordinary rectangular waveguide for which

$$Z_{0\infty} = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} \; \frac{2a}{b} \quad . \qquad \text{(eqn 4.2)}$$

When the guide width is increased at fixed slot width, the characteristic impedance at infinite frequency and the free space cutoff wavelength go asymptotically to

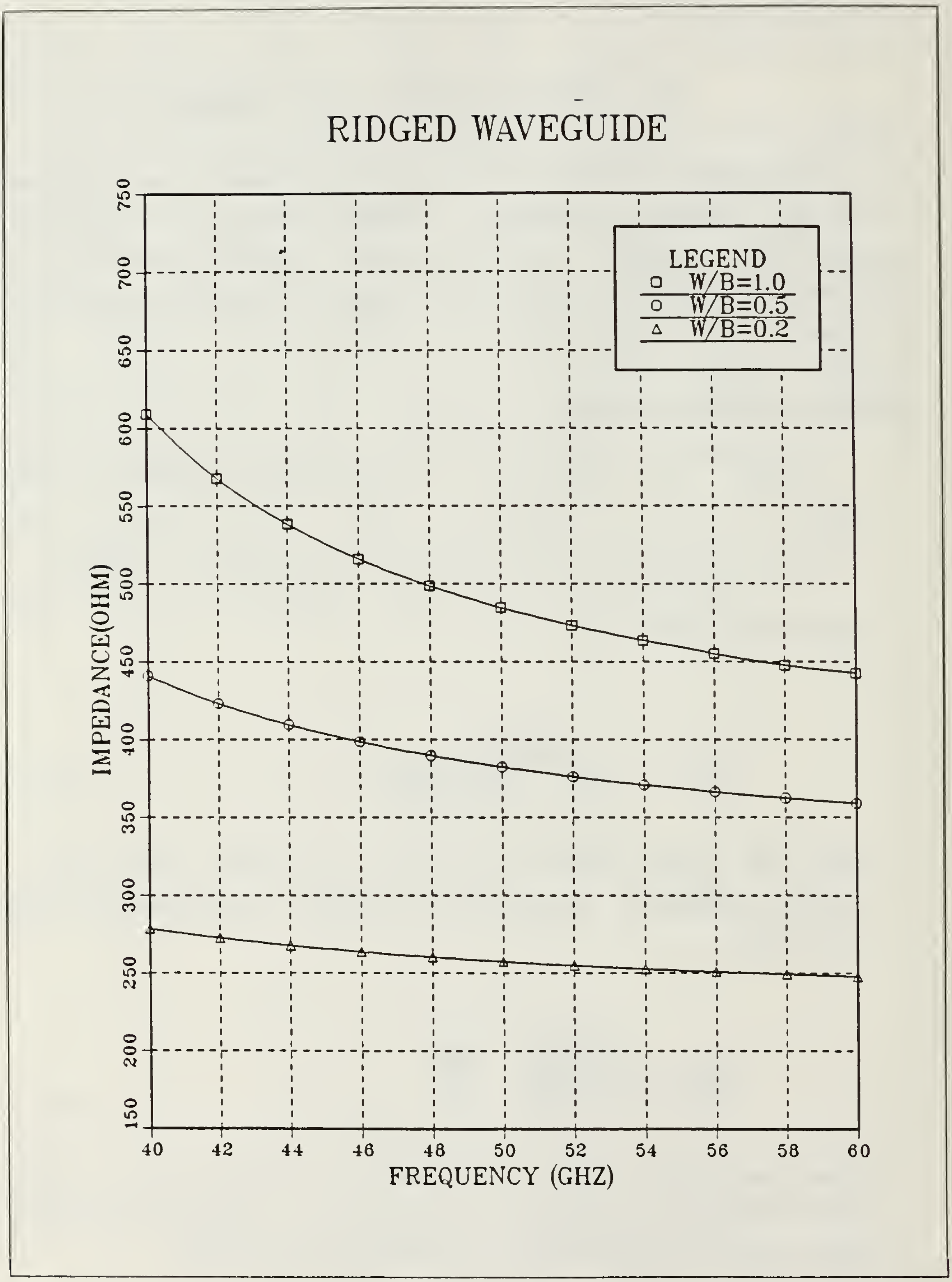

Figure 4.1 Characteristic Impedance Z vs. Frequency for a Ridged Waveguide.

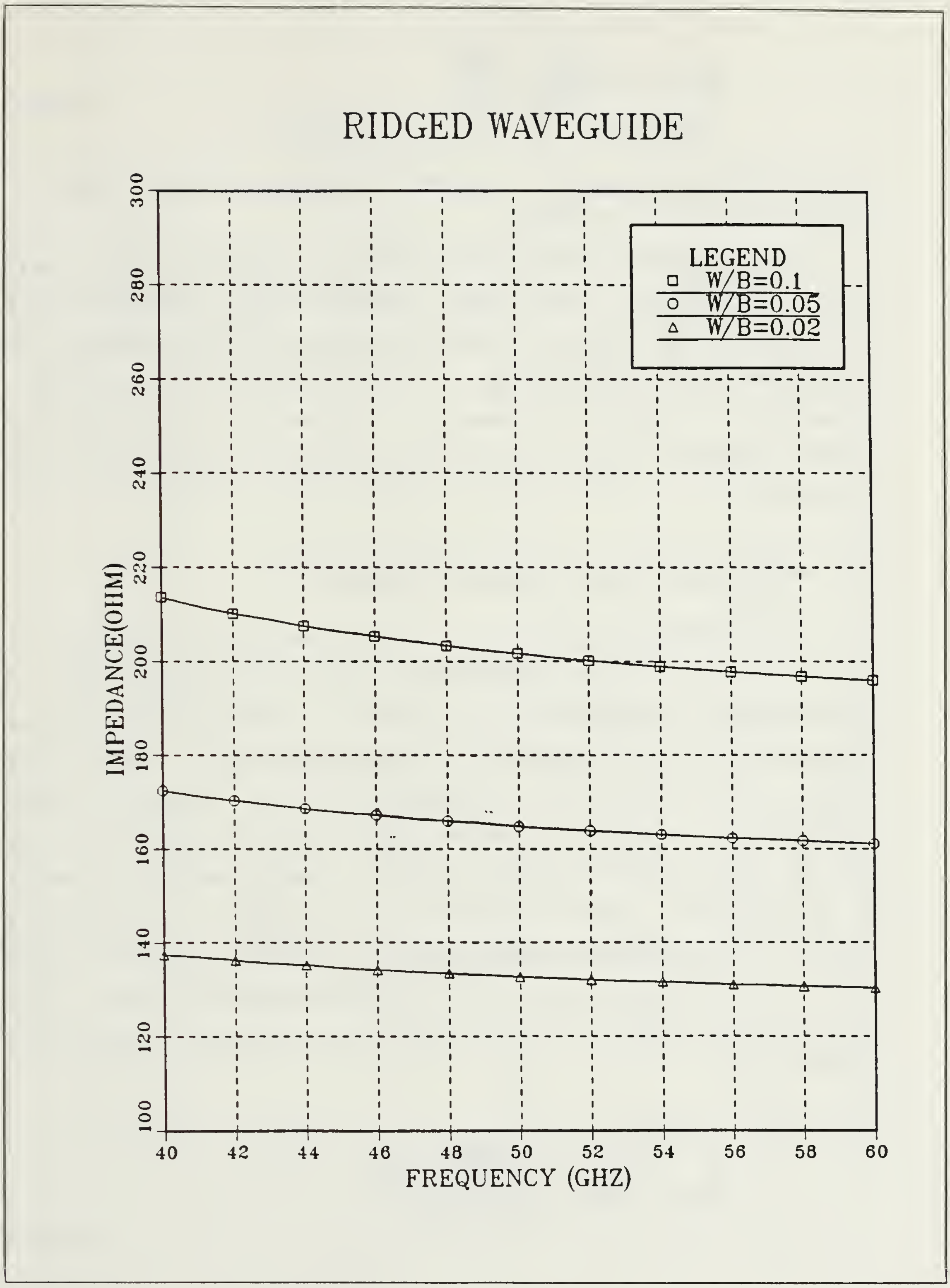

Figure 4.2 Characteristic Impedance Z vs. Frequency for a Ridged Waveguide.

$$Z_{o\infty} = \sqrt{\frac{\mu_o}{\varepsilon_r \varepsilon_o}}\, \frac{2b}{a} \quad . \qquad \text{(eqn 4.3)}$$

The cutoff wavelength λ_c was determined from [Ref. 11] for several values of W/b.

Equation (4.1) was then used to calculate the characteristic impedances and these values were compared with the results using spectral domain method of the computer program discussed in chapter III. Two results are very close together. Figure 4.1 and 4.2 shows the computer results. These figures agree with the previous Knorr and Shayda's results.

B. DIELECTRIC SLAB LOADED WAVEGUIDE

A variation of the fin-line structure of Figure 1 where w/b = 1 and ε_r > 1 results in a dielectric slab loaded rectangular waveguide. The slab loaded waveguide has also been studied by Vartanian et al [Ref. 9]. They consider a guide with the slab centered in the waveguide and they present an analytical expression for the voltage impedance Z_{pv} at the center of the slab. The impedance computed in this analysis was specified at the edge of the dielectric slab. Therefore a relationship between the two impedances must be defined before any comparison can be made. The impedance at the edge of the slab can be easily defined by the expression

$$Z_o = Z_{pv} \left(\frac{E_x^{edge}}{E_x^{center}} \right)^2 \qquad \text{(eqn 4.4)}$$

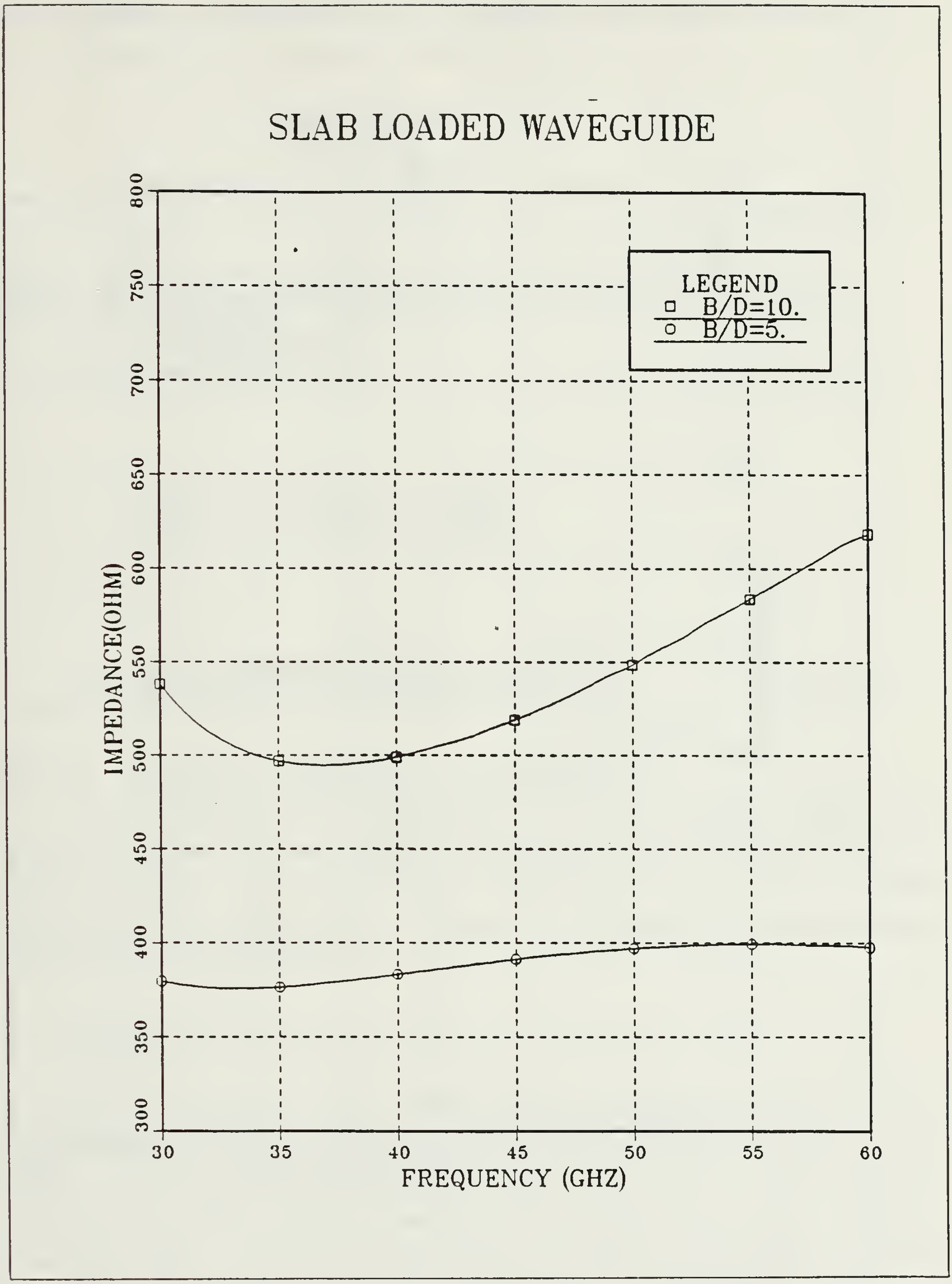

Figure 4.3 Characteristic Impedance Z vs. Frequency for a Slab Loaded Waveguide.

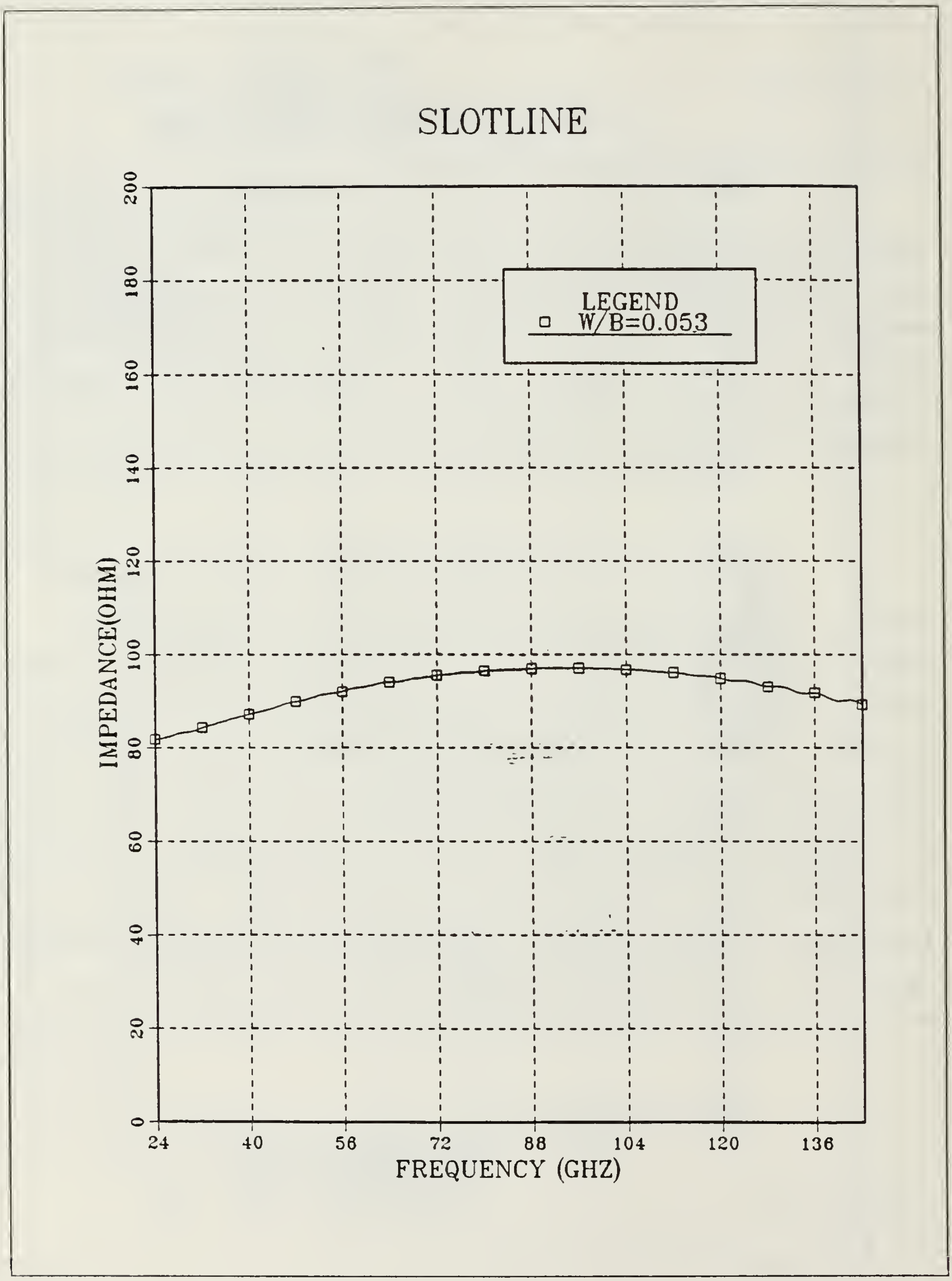

Figure 4.4 Characteristic Impedance Z vs. Frequency for a Slot Line Waveguide.

where E_x^{edge} is the field at the edge of the slab and E_x^{center} is the field at the center of slab, thus

$$Z_0 = Z_{pv} \cos^2\left(\frac{qc}{2s}\right) \qquad \text{(eqn 4.5)}$$

where the various quantities are defined in [Ref. 9] as

$$s = c/2 \qquad \text{(eqn 4.6)}$$

$$\left(\frac{q}{s}\right)^2 = \varepsilon_r\left(\frac{2\pi}{\lambda_0}\right)^2 - \left(\frac{2\pi}{\lambda_g}\right)^2 \qquad \text{(eqn 4.7)}$$

$$q^2 = (2\pi)^2\left(\frac{s}{\lambda}\right)^2\left[\varepsilon_r - \left(\frac{\lambda}{\lambda'}\right)^2\right] \qquad \text{(eqn 4.8)}$$

and c = D is the dielectric slab thickness.

The characteristic impedance of a slab loaded guide is computed using the spectral domain method. The results were compared with equation (4.5) results and [Ref. 3]. Good agreements are obtained. Figure 4.3 shows the slab loaded wave guide impedance .

C. SLOTLINE

If w/D < 2 and ε_r is sufficiently high for the fin-line structure of Figure 1.1, the field is tightly bound to the slot. For this condition the presence of the shield will have little effect if the walls are sufficiently far removed from the slot. In this case the fin-line will behave like a slotline. This behavior is illustrated in Figure 4.4 where the characteristic impedance of a fin-line with w/D = 1, ε_r = 20 have been plotted. Also these results are in good agreements with [Ref. 3].

D. SEVERAL FIN LINE IMPEDANCE CURVES

Fin-lines are generally enclosed with a shield that is compatible with the dimensions of the standard rectangular waveguides for the millimeter wavebands. Above 22GHZ, all these guides have aspect ratios b/a = 0.5. Further, the fins are most often centered in the guide and printed using D = 0.005 inch substrate with ε_r = 2.2.

Figures 4.5 - 4.12 show the impedance computed for fin-line with WR(28), WR(19) and WR(12) rectangular waveguide shields. These figures may be compared with data for the same structures as presented in [Ref. 3]. Such a comparison shows excellent agreement for all but the small values of W/b. For small values of W/b (W/b = 0.1, W/b = 0.05, W/b = 0.02), the results of the FINIMP program show lower impedances than the results presented in [Ref. 3]. As the W/b is smaller, the difference is larger; for W/b = 0.1, 2 - 5 ohm is lower, for W/b = 0.05, 9 - 15 ohm is lower and for W/b = 0.02, 19 - 25 ohm is lower.

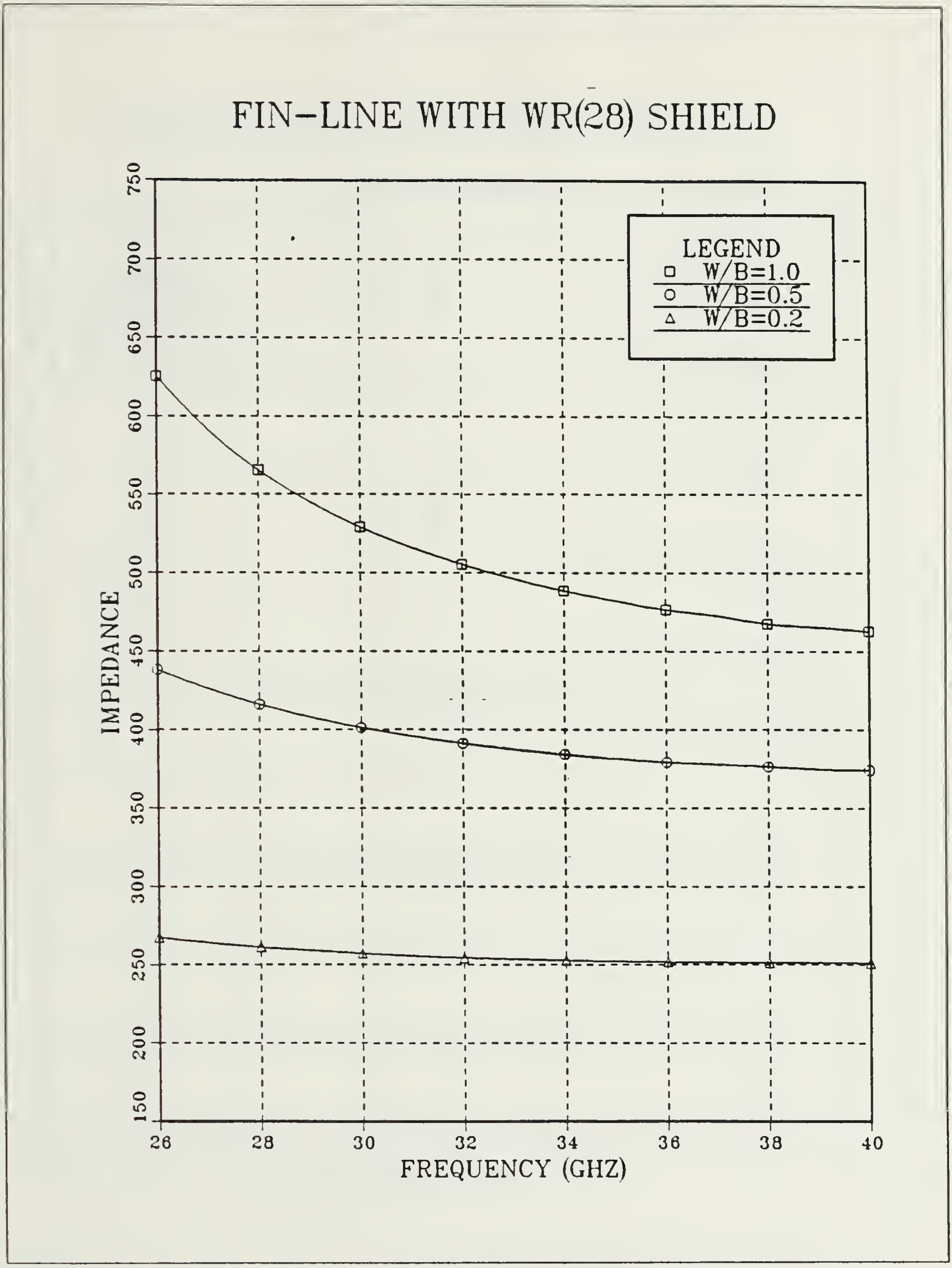

Figure 4.5 Characteristic Impedance Z vs. Frequency for a Fin-line With b/D=28.0 h_1/D=28.0 h_2/D=27.0 D=.005" ε_r=2.2.

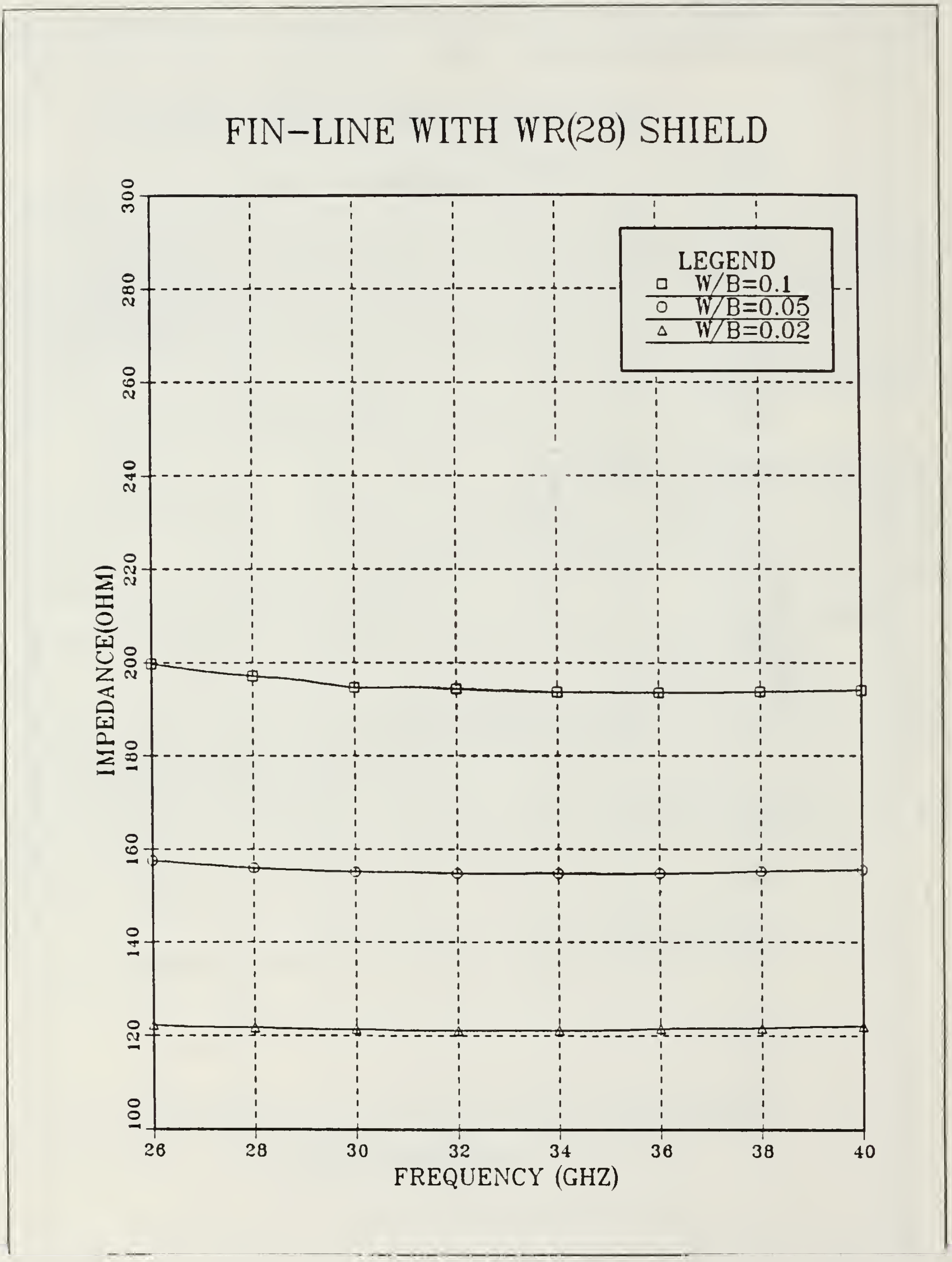

Figure 4.6 Characteris c Impedance Z vs. Frequency for a Fin-line With b D=28.0 h_1/D=28.0 h_2/D=27.0 D=.005" ε_r =2.2.

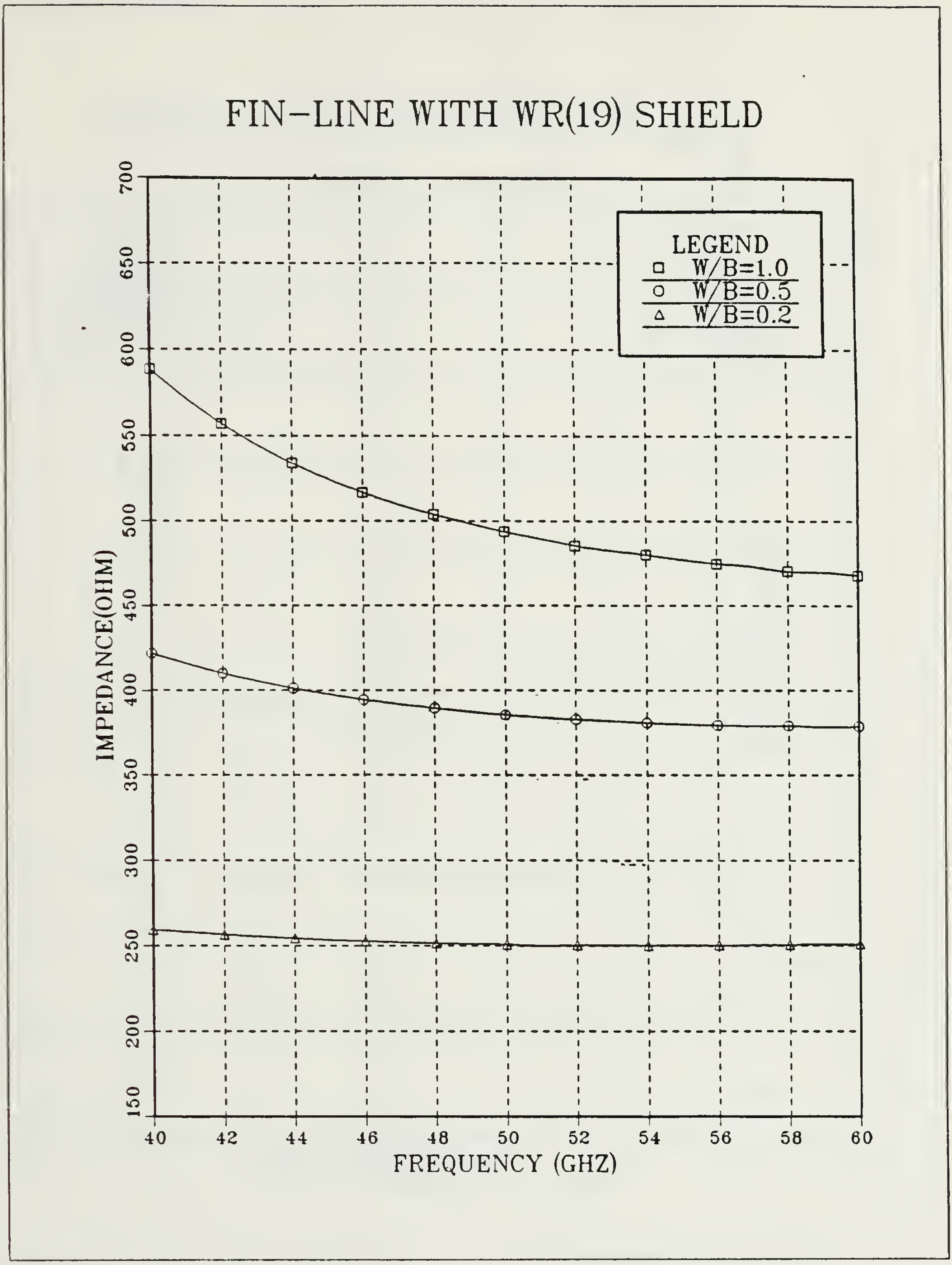

Figure 4.7 Characteristic Impedance Z vs. Frequency for a Fin-line With b/D=18.8 h_1/D=18.8 h_2/D=17.8 D=.005" ε_r =2.2.

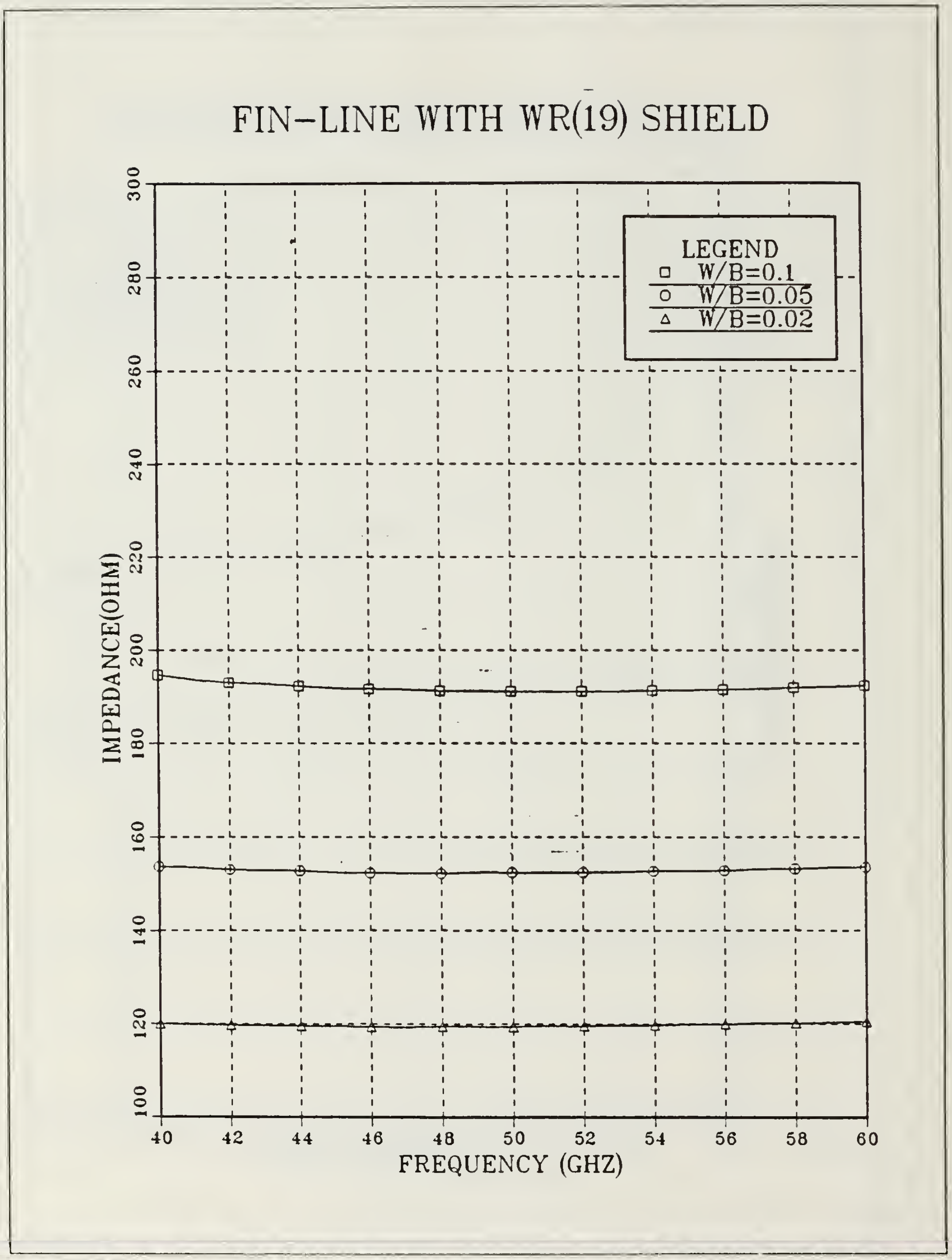

Figure 4.8 Characteristic Impedance Z vs. Frequency for a Fin-line With $b/D=18.8$ $h_1/D=18.8$ $h_2/D=17.8$ $D=.005''$ $\varepsilon_r=2.2$.

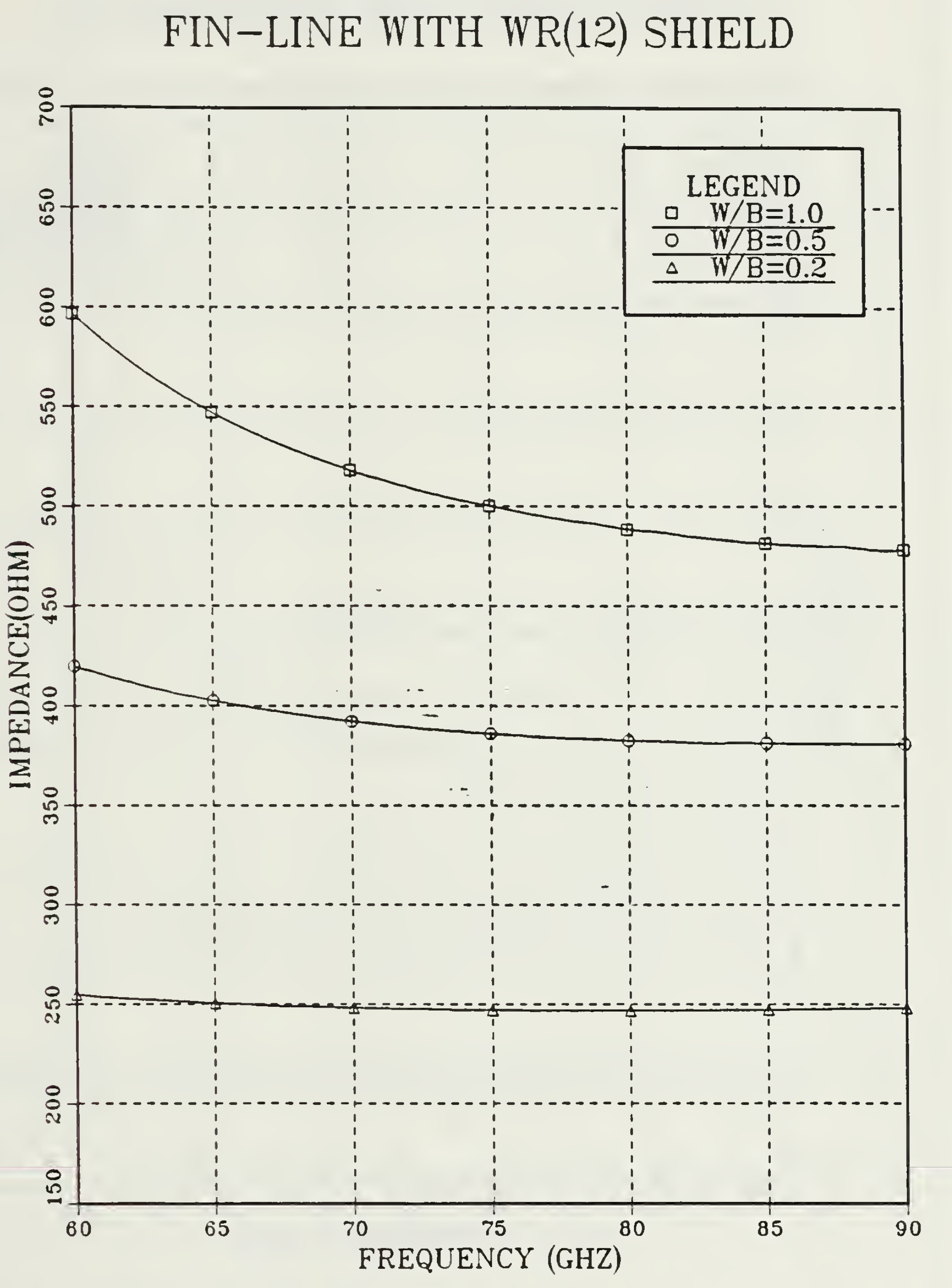

Figure 4.9 Characteristic Impedance Z vs. Frequency for a Fin-line With b/D=12.2 h_1/D=12.2 h_2/D=11.2 D=.005" ε_r =2.2.

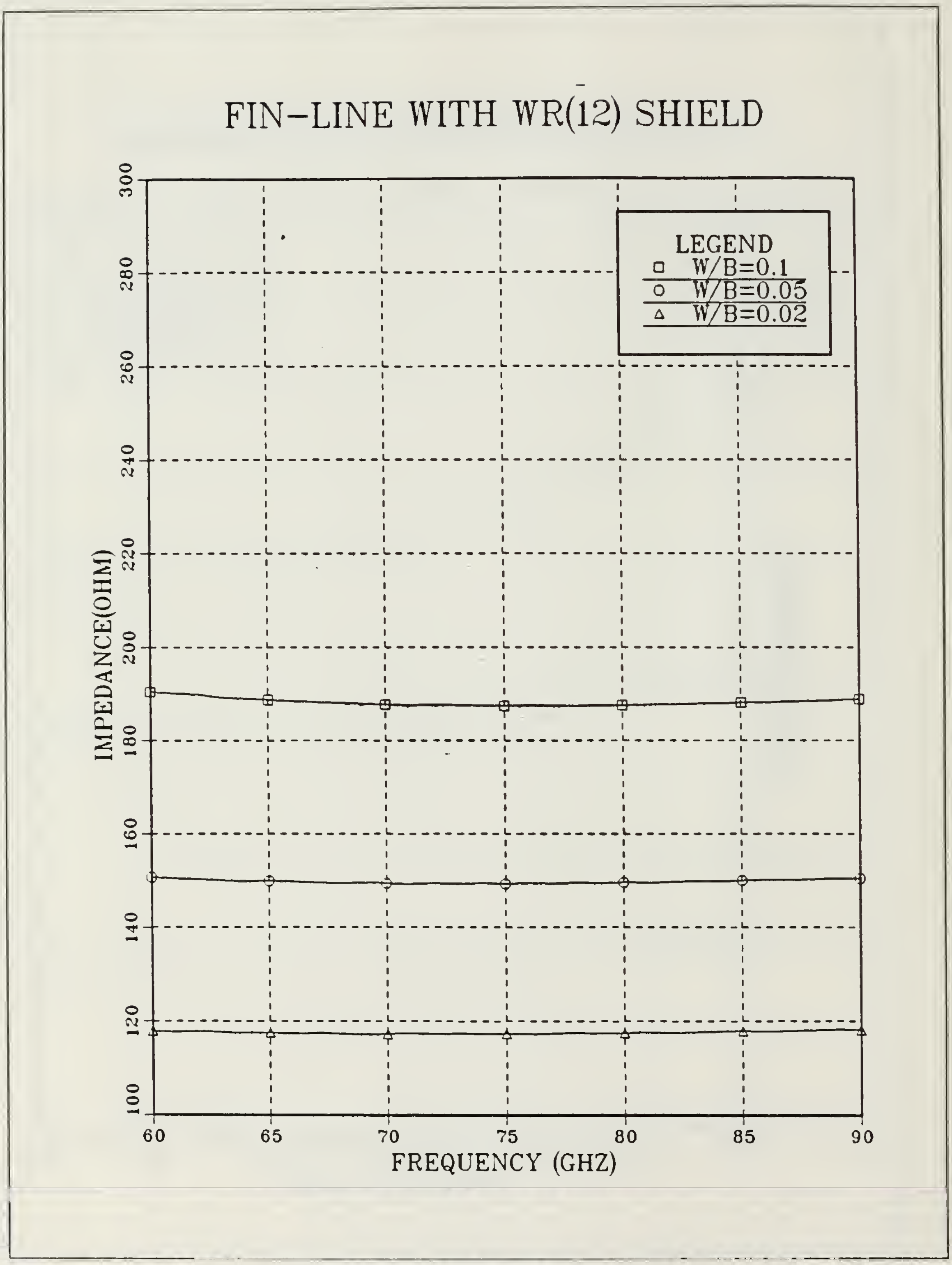

Figure 4.10 Characteristic Impedance Z vs. Frequency for a Fin-line With $b/D=12.2$ $h_1/D=12.2$ $h_2/D=11.2$ $D=.005''$ $\varepsilon_r=2.2$.

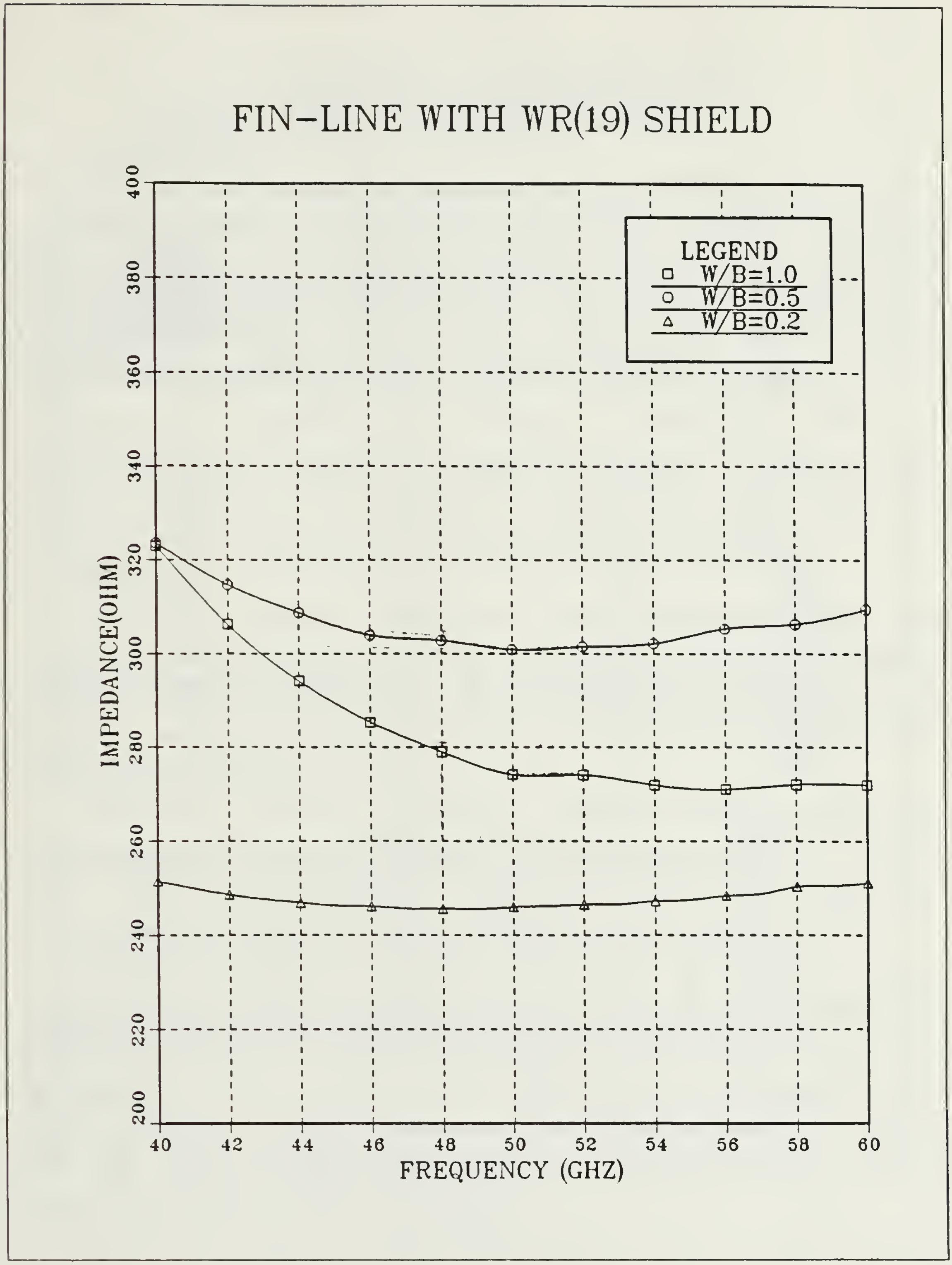

Figure 4.11 Characteristic Impedance Z vs. Frequency for a Fin-line With $b/D=18.8$ $h_1/D=28.2$ $h_2/D=8.4$ $D=.005''$ $\varepsilon_r=2.2$.

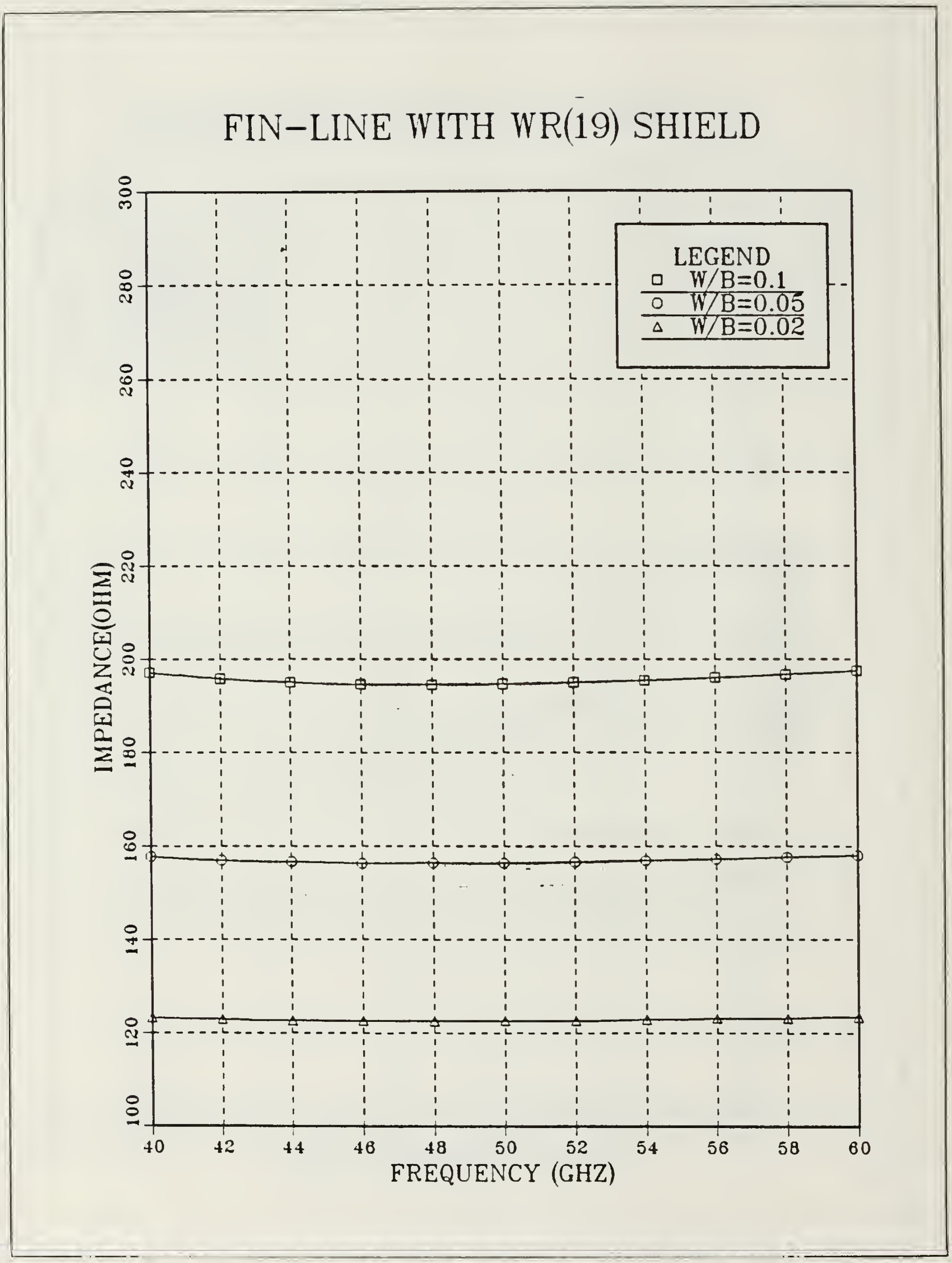

Figure 4.12 Characteristic Impedance Z vs. Frequency for a Fin-line With b/D=18.8 h_1/D=28.2 h_2/D=8.4 D=.005" ε_r =2.2.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The spectral domain technique used in conjunction with Galerkin's method has been presented to calculate the characteristic impedance for the dominant mode of the fin-line. It has been shown that a matrix formulation of the problem permits the elements of the dyadic Green's function to be calculated. Solving the matrix equations leads to containing only hyperbolic tangent function. These equations circumvent the overflow and underflow problems which occurred using the previous formulation presented by Knorr and Shayda.

Numerical results obtained using this method have been presented and compared to other existing data. Good agreement has been obtained in all cases thus establishing the accuracy and applicability of the method for the full range of structure parameters.

There is a possibility that tangent functions may cause an overflow problem if $(\gamma_i D)^2$ is less than zero. In this case, the value of tangent function can be obtained within the capability of the available IBM 3033 used.

In this thesis, paticular interest is devoted to the computation of fin-line impedance. Fin-line may exhibit the characteristics of ridged waveguide, dielectric slab loaded waveguide, slot lines, and conventional rectangular waveguide. All of these structures are fin-line substructures. So, the computation of fin-line impedance permits all of these structures to be analyzed.

B. RECOMMENDATIONS

Coupled fin-line will find future use in building directional couplers and filters. For this purpose, the normal mode wavelengths and impedances are required. A special but important case is that of symmetrical lines. Coupled fin-lines for which the normal modes are odd and even. The program described here may be extended to cover this case by following the procedures outlined by Knorr and Kuchler for coupled slotlines [Ref. 5]. This should be accomplished to improve the utility of the program described in this thesis.

APPENDIX A
SPECTRAL DOMAIN MATRICES

The continuity conditions are transformed via equation (2.24) into the two dimesional Fourier domain. The solutions to the two Helmholtz equations given by equations (2.27)-(2.32) are substituted. Finally a matrix form of linear equations is derived as follows;

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & 0 & 0 & 0 & 0 & 0 \\ m_{21} & m_{22} & m_{23} & 0 & m_{25} & m_{26} & m_{27} & 0 \\ 0 & 0 & m_{33} & m_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{43} & m_{44} & 0 & m_{46} & 0 & m_{48} \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{57} & m_{58} \\ 0 & m_{62} & 0 & m_{64} & 0 & 0 & m_{67} & m_{68} \\ m^{\varepsilon}_{71} & 0 & 0 & 0 & m^{\varepsilon}_{75} & 0 & 0 & 0 \\ m^{\varepsilon}_{81} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A^e \\ B^e \\ C^e \\ D^e \\ A^h \\ B^h \\ C^h \\ D^h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \tilde{E}_x \\ \tilde{E}_z \end{bmatrix}$$

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & 0 & 0 & 0 & 0 & 0 \\ m_{21} & m_{22} & m_{23} & 0 & m_{25} & m_{26} & m_{27} & 0 \\ 0 & 0 & m_{33} & m_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{43} & m_{44} & 0 & m_{46} & 0 & m_{48} \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{57} & m_{58} \\ 0 & m_{62} & 0 & m_{64} & 0 & 0 & m_{67} & m_{68} \\ 0 & 0 & 0 & 0 & m^{J}_{75} & m^{J}_{76} & m^{J}_{77} & 0 \\ m^{J}_{81} & m^{J}_{82} & m^{J}_{83} & 0 & m^{J}_{85} & m^{J}_{86} & m^{J}_{87} & 0 \end{bmatrix} \begin{bmatrix} A^e \\ B^e \\ C^e \\ D^e \\ A^h \\ B^h \\ C^h \\ D^h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \tilde{J}_x \\ \tilde{J}_z \end{bmatrix}$$

The matrix elements of $[M_E]$ and $[M_J]$ are normalized at this point with respect to D, the dielectric substrate thickness. The normalized matrix elements are now presented in two forms. The first of the element equations is for $(\gamma_i D)^2 > 0$ and the second is for $(\gamma_i D)^2 < 0$. For the matrices $[M_E]$ and $[M_J]$, the elements m_{11} through m_{68} are the same.

$$m_{11} = \begin{cases} (K_{c1}D)^2 \sinh\left[(\gamma_1 D)(h_1/D)\right] \\ j(K_{c1}D)^2 \sin\left[(\gamma_1'' D)(h_1/D)\right] \end{cases}$$

$$m_{12} = \begin{cases} -(K_{c2}D)^2 \sinh(\gamma_2 D) \\ -j(K_{c2}D)^2 \sin(\gamma_2'' D) \end{cases}$$

$$m_{13} = \begin{cases} -(K_{c2}D)^2 \cosh(\gamma_2 D) \\ -(K_{c2}D)^2 \cos(\gamma_2'' D) \end{cases}$$

$$m_{14} = m_{15} = m_{16} = m_{17} = m_{18} = 0$$

$$m_{21} = \begin{cases} (\alpha_n D)(\beta D) \sinh\left[(\gamma_1 D)(h_1/D)\right] \\ j(\alpha_n D)(\beta D) \sin\left[(\gamma_1'' D)(h_1/D)\right] \end{cases}$$

$$m_{22} = \begin{cases} -(\alpha_n D)(\beta D) \sinh(\gamma_2 D) \\ -j(\alpha_n D)(\beta D) \sin(\gamma_2'' D) \end{cases}$$

$$m_{23} = \begin{cases} -(\alpha_n D)(\beta D) \cosh(\gamma_2 D) \\ -(\alpha_n D)(\beta D) \cos(\gamma_2'' D) \end{cases}$$

$$m_{24} = 0$$

$$m_{25} = \begin{cases} j(\omega\mu D)(\gamma_1 D)\sinh[(\gamma_1 D)(h_1/D)] \\ -j(\omega\mu D)(\gamma_1'' D)\sin[(\gamma_1'' D)(h_1/D)] \end{cases}$$

$$m_{26} = \begin{cases} j(\omega\mu D)(\gamma_2 D)\cosh(\gamma_2 D) \\ -(\omega\mu D)(\gamma_2'' D)\cos(\gamma_2'' D) \end{cases}$$

$$m_{27} = \begin{cases} j(\omega\mu D)(\gamma_2 D)\sinh(\gamma_2 D) \\ -j(\omega\mu D)(\gamma_2'' D)\sin(\gamma_2'' D) \end{cases}$$

$$m_{28} = m_{31} = m_{32} = 0$$

$$m_{33} = \begin{cases} (K_{c2} D)^2 \\ (K_{c2} D)^2 \end{cases}$$

$$m_{34} = \begin{cases} -(K_{c3} D)^2 \sinh[(\gamma_3 D)(h_2/D)] \\ -j(K_{c3} D)^2 \sin[(\gamma_3'' D)(h_2/D)] \end{cases}$$

$$m_{35} = m_{36} = m_{37} = m_{38} = m_{41} = m_{42} = 0$$

$$m_{43} = \begin{cases} (\alpha_m D)(\beta D) \\ (\alpha_m D)(\beta D) \end{cases}$$

$$m_{44} = \begin{cases} -(\alpha_m D)(\beta D) \sinh\left[(\gamma_3 D)(h_2/D)\right] \\ -j(\alpha_m D)(\beta D) \sin\left[(\gamma_3'' D)(h_2/D)\right] \end{cases}$$

$$m_{45} = 0$$

$$m_{46} = \begin{cases} -j(\omega\mu D)(\gamma_2 D) \\ (\omega\mu D)(\gamma_2'' D) \end{cases}$$

$$m_{47} = 0$$

$$m_{48} = \begin{cases} j(\omega\mu D)(\gamma_3 D) \sinh\left[(\gamma_3 D)(h_2/D)\right] \\ -j(\omega\mu D)(\gamma_3'' D) \sin\left[(\gamma_3'' D)(h_2/D)\right] \end{cases}$$

$$m_{51} = m_{52} = m_{53} = m_{54} = m_{55} = m_{56} = 0$$

$$m_{57} = \begin{cases} (K_{c2} D)^2 \\ (K_{c2} D)^2 \end{cases}$$

$$m_{58} = \begin{cases} -(K_{c3}D)^2 \cosh\left[(\gamma_3 D)(h_2/D)\right] \\ -(K_{c3}D)^2 \cos\left[(\gamma_3'' D)(h_2/D)\right] \end{cases}$$

$$m_{61} = 0$$

$$m_{62} = \begin{cases} j(\omega\varepsilon_2 D)(\gamma_2 D) \\ -j(\omega\varepsilon_2 D)(\gamma_2'' D) \end{cases}$$

$$m_{63} = 0$$

$$m_{64} = \begin{cases} -j(\omega\varepsilon_3 D)(\gamma_3 D)\cosh\left[(\gamma_3 D)(h_2/D)\right] \\ (\omega\varepsilon_3 D)(\gamma_3'' D)\cos\left[(\gamma_3'' D)(h_2/D)\right] \end{cases}$$

$$m_{65} = m_{66} = 0$$

$$m_{67} = \begin{cases} (\alpha_m D)(\beta D) \\ (\alpha_m D)(\beta D) \end{cases}$$

$$m_{68} = \begin{cases} -(\alpha_m D)(\beta D)\cosh\left[(\gamma_3 D)(h_2/D)\right] \\ -(\alpha_m D)(\beta D)\cos\left[(\gamma_3'' D)(h_2/D)\right] \end{cases}$$

$$m_{71}^{\varepsilon} = \begin{cases} (\alpha_m D)(\beta D)\sinh[(\gamma_1 D)(h_1/D)] \\ j(\alpha_m D)(\beta D)\sin[(\gamma_1'' D)(h_1/D)] \end{cases}$$

$$m_{72}^{\varepsilon} = m_{73}^{\varepsilon} = m_{74}^{\varepsilon} = 0$$

$$m_{75}^{\varepsilon} = \begin{cases} j(\omega\mu D)(\gamma_1 D)\sinh[(\gamma_1 D)(h_1/D)] \\ -j(\omega\mu D)(\gamma_1'' D)\sin[(\gamma_1'' D)(h_1/D)] \end{cases}$$

$$m_{76}^{\varepsilon} = m_{77}^{\varepsilon} = m_{78}^{\varepsilon} = 0$$

$$m_{81}^{\varepsilon} = \begin{cases} (k_{c1} D)^2 \sinh[(\gamma_1 D)(h_1/D)] \\ j(k_{c1} D)^2 \sin[(\gamma_1'' D)(h_1/D)] \end{cases}$$

$$m_{82}^{\varepsilon} = m_{83}^{\varepsilon} = m_{84}^{\varepsilon} = m_{85}^{\varepsilon} = m_{86}^{\varepsilon} = m_{87}^{\varepsilon} = m_{88}^{\varepsilon} = 0$$

$$m_{71}^{J} = m_{72}^{J} = m_{73}^{J} = m_{74}^{J} = 0$$

$$m_{75}^{J} = \begin{cases} (k_{c1} D)^2 \cosh[(\gamma_1 D)(h_1/D)] \\ (k_{c1} D)^2 \cos[(\gamma_1'' D)(h_1/D)] \end{cases}$$

$$m_{76}^{J} = \begin{cases} -(K_{c2}D)^2 \sinh(\gamma_2 D) \\ -j(K_{c2}D)^2 \sin(\gamma_2'' D) \end{cases}$$

$$m_{77}^{J} = \begin{cases} -(K_{c2}D)^2 \cosh(\gamma_2 D) \\ -(K_{c2}D)^2 \cos(\gamma_2'' D) \end{cases}$$

$$m_{78}^{J} = 0$$

$$m_{81}^{J} = \begin{cases} -j(\omega\varepsilon_1 D)(\gamma_1 D) \cosh[(\gamma_1 D)(h_1/D)] \\ (\omega\varepsilon_1 D)(\gamma_1'' D) \cos[(\gamma_1'' D)(h_1/D)] \end{cases}$$

$$m_{82}^{J} = \begin{cases} -j(\omega\varepsilon_2 D)(\gamma_2 D) \cosh(\gamma_2 D) \\ (\omega\varepsilon_2 D)(\gamma_2'' D) \cos(\gamma_2'' D) \end{cases}$$

$$m_{83}^{J} = \begin{cases} -j(\omega\varepsilon_2 D)(\gamma_2 D) \sinh(\gamma_2 D) \\ j(\omega\varepsilon_2 D)(\gamma_2'' D) \sin(\gamma_2'' D) \end{cases}$$

$$m_{84}^{J} = 0$$

$$m_{85}^{J} = \begin{cases} (\alpha_m D)(\beta D) \cosh[(\gamma_1 D)(h_1/D)] \\ (\alpha_m D)(\beta D) \cos[(\gamma_1'' D)(h_1/D)] \end{cases}$$

$$m_{86}^{J} = \begin{cases} (\alpha_m D)(\beta D)\cosh\left[(\gamma_1 D)(h_1/D)\right] \\ (\alpha_m D)(\beta D)\cos\left[(\gamma_1' D)(h_1/D)\right] \end{cases}$$

$$m_{87}^{J} = \begin{cases} -(\alpha_m D)(\beta D)\sinh(\gamma_2 D) \\ -j(\alpha_m D)(\beta D)\sin(\gamma_2'' D) \end{cases}$$

$$m_{88}^{J} = 0$$

APPENDIX B

TIME AVERAGE POWER FLOW

The following expression for the coefficients A^e through D^h are derived from equations (2.54) - (2.61).

If $(\gamma_1 D)^2 < 0$

$$A^e = -j\left[\frac{D^2\tilde{E}_z}{(K_{c1}D)^2 \sin[(\gamma_1'' D)(h_1/D)]}\right]$$

$$A^h = j\left[\frac{(K_{c1}D)^2 D^2\tilde{E}_x - (\alpha_m D)(\beta D)D^2\tilde{E}_z}{(\omega\mu D)(K_{c1}D)^2(\gamma_1'' D)\sin[(\gamma_1'' D)(h_1/D)]}\right]$$

If $(\gamma_1 D)^2 > 0$

$$A^e = \frac{D^2 E_z}{(k_{c1}D)^2 \sinh[(\gamma_1 D)(h_1/D)]}$$

$$A^h = -j\left[\frac{(K_{c1}D)^2 D^2\tilde{E}_x - (\alpha_m D)(\beta D)D^2 E_z}{(\omega\mu D)(\gamma_1 D)(K_{c1}D)^2 \sinh[(\gamma_1 D)(h_1/D)]}\right]$$

If $(\gamma_2 D)^2 < 0$

$$C^e = \frac{d_{12}D^2\tilde{E}_x - d_{22}D^2\tilde{E}_z}{\det \cos(\gamma_2'' D)}$$

$$C^h = j\left[\frac{d_{21}D^2 E_z - d_{11}D^2\tilde{E}_x}{\det (\gamma_2'' D)\sin(\gamma_2'' D)}\right]$$

where det = $d_{11}\, d_{22} - d_{21}\, d_{12}$

If $(\gamma_2 D)^2 > 0$

$$C^e = \frac{d_{12} D^2 \tilde{E}_x - d_{22} D^2 E_z}{\det \cosh(\gamma_2 D)}$$

$$C^h = j \left[\frac{d_{11} D^2 \hat{\tilde{E}}_x - d_{21} D^2 \hat{\tilde{E}}_z}{\det (\gamma_2 D) \sinh(\gamma_2 D)} \right]$$

If $(\gamma_2 D)^2 < 0$ and $(\gamma_3 D)^2 < 0$

$$B^e = -j \left[\frac{(\omega \varepsilon_3 D)(\gamma_3'' D)(K_{c2} D)^2}{(\omega \varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 \tan[(\gamma_3'' D)(h_2/D)]} \right] \left[\frac{d_{12} D^2 \hat{\tilde{E}}_x - d_{22} D^2 \hat{\tilde{E}}_z}{\det \cos(\gamma_2'' D)} \right]$$

$$+ j \left[\frac{(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega \mu D)(\gamma_2'' D)(K_{c3} D)^2} \right] \left[\frac{d_{11} D^2 \hat{\tilde{E}}_x - d_{21} D^2 \hat{\tilde{E}}_z}{\det (\gamma_2'' D) \sin(\gamma_2'' D)} \right]$$

$$B^h = \left[\frac{(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega \mu D)(\gamma_2'' D)(K_{c3} D)^2} \right] \left[\frac{d_{12} D^2 \hat{\tilde{E}}_x - d_{22} D^2 \hat{\tilde{E}}_z}{\det \cos(\gamma_2'' D)} \right]$$

$$+ \left[\frac{(\gamma_3'' D)(K_{c2} D)^2 \tan[(\gamma_3'' D)(h_2/D)]}{(\gamma_2'' D)(K_{c2} D)^2} \right] \left[\frac{d_{11} D^2 \tilde{E}_x - d_{21} D^2 \tilde{E}_z}{\det (\gamma_2'' D) \sin(\gamma_2'' D)} \right]$$

$$D^e = -j \left[\frac{(K_{c2} D)^2 (d_{12} D^2 \hat{\tilde{E}}_x - d_{22} D^2 \hat{\tilde{E}}_z)}{\det \cos(\gamma_2'' D)(K_{c3} D)^2 \sin[(\gamma_3'' D)(h_2/D)]} \right]$$

$$D^h = -j\left[\frac{(K_{c2}D)^2(d_{11}D^2\hat{E}_x - d_{21}D^2\hat{E}_z)}{\det(\gamma_2''D)\sin(\gamma_2''D)(K_{c3}D)^2\cos[(\gamma_3''D)(h_2/D)]}\right]$$

If $(\gamma_2 D)^2 < 0$ and $(\gamma_3 D)^2 > 0$

$$B^e = -j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2''D)(K_{c3}D)^2\tanh[(\gamma_3 D)(h_2/D)]}\right]\left[\frac{d_{12}D^2\hat{E}_x - d_{22}D^2\tilde{E}_z}{\det\cos(\gamma_2''D)}\right]$$

$$+j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{d_{11}D^2\hat{E}_x - d_{21}D^2\tilde{E}_z}{\det(\gamma_2''D)\sin(\gamma_2''D)}\right]$$

$$B^h = \left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\mu D)(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{d_{12}D^2E_x - d_{22}D^2E_z}{\det\cos(\gamma_2''D)}\right]$$

$$-\left[\frac{(\gamma_3 D)(K_{c2}D)^2\tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{d_{11}D^2E_x - d_{21}D^2E_z}{\det(\gamma_2''D)\sin(\gamma_2''D)}\right]$$

$$D^e = \frac{(K_{c2}D)^2(d_{12}D^2\tilde{E}_x - d_{22}D^2\hat{E}_z)}{\det\cos(\gamma_2''D)(K_{c3}D)^2\sinh[(\gamma_3 D)(h_2/D)]}$$

$$D^h = -j\left[\frac{(K_{c2}D)^2(d_{11}D^2E_x - d_{21}D^2E_z)}{\det(\gamma_2''D)\sin(\gamma_2''D)(K_{c3}D)^2\cosh[(\gamma_3 D)(h_2/D)]}\right]$$

If $(\gamma_2 D)^2 > 0$ and $(\gamma_3 D)^2 < 0$

$$B^e = \left[\frac{(\omega\epsilon_3 D)(\gamma_3'' D)(K_{c2}D)^2}{(\omega\epsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tan[(\gamma_3'' D)(h_2/D)]}\right]\left[\frac{d_{12}D^2\tilde{E}_x - d_{22}D^2\tilde{E}_z}{\det \cosh(\gamma_2 D)}\right]$$

$$+\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\epsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{d_{11}D^2\tilde{E}_x - d_{21}D^2\tilde{E}_z}{\det(\gamma_2 D)\sinh(\gamma_2 D)}\right]$$

$$B^h = j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{d_{12}D^2\tilde{E}_x - d_{22}D^2\tilde{E}_z}{\det \cosh(\gamma_2 D)}\right]$$

$$-j\left[\frac{(\gamma_3'' D)(K_{c2}D)^2 \tan[(\gamma_3'' D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{d_{11}D^2\tilde{E}_x - d_{21}D^2\tilde{E}_z}{\det(\gamma_2 D)\sinh(\gamma_2 D)}\right]$$

$$D^e = -j\left[\frac{(K_{c2}D)^2(d_{11}D^2\tilde{E}_x - d_{21}D^2\tilde{E}_z)}{\det \cosh(\gamma_2 D)(K_{c3}D)^2 \sin[(\gamma_3'' D)(h_2/D)]}\right]$$

$$D^h = j\left[\frac{(K_{c2}D)^2(d_{11}D^2\tilde{E}_x - d_{21}D^2\tilde{E}_z)}{\det(\gamma_2 D)\sinh(\gamma_2 D)(K_{c3}D)^2 \cos[(\gamma_3'' D)(h_2/D)]}\right]$$

If $(\gamma_2 D)^2 > 0$ and $(\gamma_3 D)^2 > 0$

$$B^e = \left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right]\left[\frac{d_{12}D^2\tilde{E}_x - d_{22}D^2\tilde{E}_z}{\det\cosh(\gamma_2 D)}\right]$$

$$+\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{d_{11}D^2\tilde{E}_x - d_{21}D^2\tilde{E}_z}{\det(\gamma_2 D)\sinh(\gamma_2 D)}\right]$$

$$B^h = j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{d_{12}D^2\tilde{E}_x - d_{22}D^2\tilde{E}_z}{\det\cosh(\gamma_2 D)}\right]$$

$$+j\left[\frac{(\gamma_3 D)(K_{c2}D)^2\tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{d_{11}D^2\tilde{E}_x - d_{21}D^2\tilde{E}_z}{\det(\gamma_2 D)\sinh(\gamma_2 D)}\right]$$

$$D^e = \frac{(K_{c2}D)^2(d_{12}D^2\tilde{E}_x - d_{22}D^2\tilde{E}_z)}{\det\cosh(\gamma_2 D)(K_{c3}D)^2\sinh[(\gamma_3 D)(h_2/D)]}$$

$$D^h = j\left[\frac{(K_{c2}D)^2(d_{11}D^2 E_x - d_{21}D^2 E_z)}{\det(\gamma_2 D)\sinh(\gamma_2 D)(K_{c3}D)^2\cosh[(\gamma_3 D)(h_2/D)]}\right]$$

From the above equations

If $(\gamma_1 D)^2 < 0$

$$\frac{A^e\cdot(A^e)^*}{D^4} = \frac{\tilde{E}_z^2\left[1+\tan^2[(\gamma_1'' D)^2(h_1/D)]\right]}{(K_{c1}D)^4\tan^2[(\gamma_1'' D)(h_1/D)]}$$

$$\frac{A^h \cdot (A^h)^*}{D^4} = \frac{[(K_{c1}D)^2 \tilde{E}_x - (\alpha_m D)(\beta D)\tilde{E}_z]^2 [1 + \tan^2[(\gamma_1'' D)(h_1/D)]]}{(\omega \mu D)^2 (\gamma_1'' D)^2 (K_{c1}D)^4 \tan[(\gamma_1'' D)(h_1/D)]}$$

$$\frac{A^e \cdot (A^h)^*}{D^4} = -\frac{\tilde{E}_z [(K_{c1}D)^2 E_x - (\alpha_m D)(\beta D)\tilde{E}_z [1 + \tan^2[(\gamma_1'' D)(h_1/D)]]}{(\omega \mu D)(\gamma_1'' D)(K_{c1}D)^4 \tan^2[(\gamma_1'' D)(h_1/D)]}$$

$$\frac{(A^e)^* \cdot A^h}{D^4} = \frac{A^e \cdot (A^h)^*}{D^4}$$

If $(\gamma_1 D)^2 > 0$

$$\frac{A^e \cdot (A^e)^*}{D^4} = \frac{\tilde{E}_z^2 [1 - \tanh^2[(\gamma_1 D)(h_1/D)]]}{(K_{c1}D)^4 \tanh^2[(\gamma_1 D)(h_1/D)]}$$

$$\frac{A^h \cdot (A^h)^*}{D^4} = \frac{[(K_{c1}D)^2 \tilde{E}_x - (\alpha_m D)(\beta D)\tilde{E}_z]^2 [1 - \tanh^2[(\gamma_1 D)(h_1/D)]]}{(\omega \mu D)^2 (\gamma_1 D)^2 (K_{c1}D)^4 \tanh^2[(\gamma_1 D)(h_1/D)]}$$

$$\frac{A^e \cdot (A^h)^*}{D^4} = j \left[\frac{\tilde{E}_z [(K_{c1}D)^2 \tilde{E}_x - (\alpha_m D)(\beta D) E_z][1 - \tanh^2[(\gamma_1 D)(h_1/D)]]}{(K_{c1}D)^4 (\omega \mu D)(\gamma_1 D) \tanh^2[(\gamma_1 D)(h_1/D)]} \right]$$

$$\frac{(A^e)^* \cdot A^h}{D^4} = -\frac{A^e \cdot (A^h)^*}{D^4}$$

If $(\gamma_2 D)^2 < 0$

$$\frac{c^e \cdot (c^e)^*}{D^4} = \left[\frac{d_{12}\tilde{E}_x - d_{22}\tilde{E}_z}{\det}\right]^2 \left[1 + \tan^2(\gamma_2'' D)\right]$$

$$\frac{c^h \cdot (c^h)^*}{D^4} = \left[\frac{d_{11}\tilde{E}_x - d_{21}\tilde{E}_z}{\det(\gamma_2'' D)}\right]^2 \left[\frac{1 + \tan^2(\gamma_2'' D)}{\tan^2(\gamma_2'' D)}\right]$$

$$\frac{c^e \cdot (c^h)^*}{D^4} = j\left[\frac{(d_{12}\tilde{E}_x - d_{22}\tilde{E}_z)(d_{11}\tilde{E}_x - d_{21}\tilde{E}_z)}{\det^2}\right]\left[\frac{1 + \tan^2(\gamma_2'' D)}{(\gamma_2'' D)\tan(\gamma_2'' D)}\right]$$

$$\frac{(c^e)^* \cdot c^h}{D^4} = -\frac{c^e \cdot (c^h)^*}{D^4}$$

If $(\gamma_2 D)^2 > 0$

$$\frac{c^e \cdot (c^e)^*}{D^4} = \left[\frac{d_{12}\tilde{E}_x - d_{21}\tilde{E}_z}{\det(\gamma_2 D)}\right]^2 \left[1 - \tanh^2(\gamma_2 D)\right]$$

$$\frac{c^h \cdot (c^h)^*}{D^4} = \left[\frac{d_{11}\tilde{E}_x - d_{21}\tilde{E}_z}{\det(\gamma_2 D)}\right]^2 \left[\frac{1 - \tanh^2(\gamma_2 D)}{\tanh^2(\gamma_2 D)}\right]$$

$$\frac{c^e \cdot (c^h)^*}{D^4} = -j\left[\frac{(d_{12}\tilde{E}_x - d_{22}\tilde{E}_z)(d_{11}\tilde{E}_x - d_{21}\tilde{E}_z)}{\det^2}\right]\left[\frac{1 - \tanh^2(\gamma_2 D)}{(\gamma_2 D)\tanh(\gamma_2 D)}\right]$$

$$\frac{(C^e)^* \cdot C^h}{D^4} = -\frac{C^e \cdot (C^h)^*}{D^4}$$

If $(\gamma_2 D)^2 < 0$ and $(\gamma_3 D)^2 < 0$

$$\frac{B^e \cdot (B^e)^*}{D^4} = \left[\frac{(\omega\varepsilon_3 D)(\gamma_3'' D)(K_{c2} D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 (\gamma_3'' D) \tan[(\gamma_3'' D)(h_2/D)]}\right]^2 \left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$-2\left[\frac{(\omega\varepsilon_3 D)(\gamma_3'' D)(K_{c2} D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 (\gamma_3'' D) \tan[(\gamma_3'' D)(h_2/D)]}\right] \cdot$$

$$\left[\frac{(\alpha_n D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2}\right]\left[\frac{C^e \cdot (C^h)^*}{D^4}\right]$$

$$+\left[\frac{(\alpha_n D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2}\right]^2 \left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{B^h \cdot (B^h)^*}{D^4} = \left[\frac{(\alpha_n D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\mu D)(\gamma_2'' D)(K_{c3} D)^2}\right]^2 \left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$-2\left[\frac{(\alpha_n D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\mu D)(\gamma_2'' D)(K_{c3} D)^2}\right] \cdot$$

$$\left[\frac{(\gamma_3''D)(K_{c2}D)^2 \tan[(\gamma_3''D)(h_2/D)]}{(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{D^4}\right]$$

$$+\left[\frac{(\gamma_3''D)(K_{c2}D)^2 \tan[(\gamma_3''D)(h_2/D)]}{(\gamma_2''D)(K_{c3}D)^2}\right]^2\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{B^e \cdot (B^h)^*}{D^4} = -j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3''D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2''D)(K_{c3}D)^2 \tan[(\gamma_3''D)(h_2/D)]}\right]\cdot$$

$$\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\mu D)(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3''D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{(\gamma_3''D)(K_{c2}D)^2}{(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{jD^4}\right]$$

$$+j\left[\frac{(\alpha_m D)^2(\beta D)^2[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\omega\mu D)(\gamma_2''D)(K_{c3}D)^4}\right]\left[\frac{C^e \cdot (C^h)^*}{jD^4}\right]$$

$$+j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\gamma_2''D)(K_{c3}D)^2}\right]\cdot$$

$$\left[\frac{(\gamma_3''D)(K_{c2}D)^2 \tan[(\gamma_3''D)(h_2/D)]}{(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{(B^e)^* \cdot B^h}{D^4} = -\frac{B^e \cdot (B^h)^*}{D^4}$$

$$\frac{B^e \cdot (C^h)^*}{D^4} = \left[\frac{(\omega\varepsilon_3 D)(\gamma_3'' D)(K_{c2} D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 \tan\left[(\gamma_3'' D)(h_2/D)\right]}\right]\left[\frac{C^e \cdot (C^h)^*}{j D^4}\right]$$

$$-\left[\frac{(\alpha_m D)(\beta D)\left[(K_{c2} D)^2 - (K_{c3} D)^2\right]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2}\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{(B^e)^* \cdot C^h}{D^4} = \frac{B^e \cdot (C^h)^*}{D^4}$$

$$\frac{B^e \cdot (C^e)^*}{D^4} = -j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3'' D)(K_{c2} D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 \tan\left[(\gamma_3'' D)(h_2/D)\right]}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$-j\left[\frac{(\alpha_m D)(\beta D)\left[(K_{c2} D)^2 - (K_{c3} D)^2\right]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{D^4}\right]$$

$$\frac{(B^e)^* \cdot C^e}{D^4} = -\frac{B^e \cdot (C^e)^*}{D^4}$$

$$\frac{B^h \cdot (C^e)^*}{D^4} = \left[\frac{(\alpha_m D)(\beta D)\left[(K_{c2} D)^2 - (K_{c3} D)^2\right]}{(\omega\mu D)(\gamma_2'' D)(K_{c3} D)^2}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+\left[\frac{(\gamma_3'D)(K_{c2}D)^2\tan[(\gamma_3'D)(h_2/D)]}{(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{C^e\cdot(C^h)^*}{jD^4}\right]$$

$$\frac{(B^h)^*\cdot C^e}{D^4} = \frac{B^h\cdot(C^e)^*}{D^4}$$

$$\frac{B^h\cdot(C^h)^*}{D^4} = j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\mu D)(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{C^e\cdot(C^h)^*}{jD^4}\right]$$

$$+j\left[\frac{(\gamma_3'D)(K_{c2}D)^2\tan[(\gamma_3''D)(h_2/D)]}{(\gamma_2''D)(K_{c3}D)^2}\right]\left[\frac{C^h\cdot(C^h)^*}{D^4}\right]$$

$$\frac{(B^h)^*\cdot C^h}{D^4} = -\frac{B^h\cdot(C^h)^*}{D^4}$$

$$\frac{D^e\cdot(D^e)^*}{D^4} = \left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2\left[\frac{1+\tan^2[(\gamma_3''D)(h_2/D)]}{\tan^2[(\gamma_3''D)(h_2/D)]}\right]\left[\frac{C^e\cdot(C^e)^*}{D^4}\right]$$

$$\frac{D^h\cdot(D^h)^*}{D^4} = \left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2\left[1+\tan^2[(\gamma_3''D)(h_2/D)]\right]\left[\frac{C^h\cdot(C^h)^*}{D^4}\right]$$

$$\frac{D^e\cdot(D^h)^*}{D^4} = \left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2\left[\frac{1+\tan^2[(\gamma_3'D)(h_2/D)]}{\tan[(\gamma_3''D)(h_2/D)]}\right]\left[\frac{C^e\cdot(C^h)^*}{jD^4}\right]$$

$$\frac{(D^e)^* \cdot D^h}{D^4} = \frac{D^e \cdot (D^h)^*}{D^4}$$

If $(\gamma_2 D)^2 < 0$ and $(\gamma_3 D)^2 > 0$

$$\frac{B^e \cdot (B^e)^*}{D^4} = \left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2} D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right]^2 \left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$-2\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2} D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right] \cdot$$

$$\left[\frac{(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2}\right]\left[\frac{C^e \cdot (C^h)^*}{D^4}\right]$$

$$+\left[\frac{(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2}\right]^2 \left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{B^h \cdot (B^h)^*}{D^4} = \left[\frac{(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\mu D)(\gamma_2'' D)(K_{c3} D)^2}\right]^2 \left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+\left[\frac{2(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\mu D)(\gamma_2'' D)(K_{c3} D)^2}\right] \cdot$$

$$\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2'' D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{j D^4}\right]$$

$$+\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2'' D)(K_{c3}D)^2}\right]^2\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{B^e \cdot (B^h)^*}{D^4} = -j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right]\cdot$$

$$\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\mu D)(\gamma_2'' D)(K_{c3}D)^2}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3}D)^2}\right]\left[\frac{(\gamma_3 D)(K_{c2}D)^2}{(\gamma_2'' D)(K_{c3}D)^2}\right]\left[\frac{C^e \cdot (C^h)^*}{j D^4}\right]$$

$$+j\left[\frac{(\alpha_m D)^2(\beta D)^2[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\omega\mu D)(\gamma_2'' D)(K_{c3}D)^4}\right]\left[\frac{C^e \cdot (C^h)^*}{j D^4}\right]$$

$$-j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3}D)^2}\right]\cdot$$

$$\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2'' D)(K_{c3}D)^2}\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{(B^e)^* \cdot B^h}{D^4} = -\frac{B^e \cdot (B^h)^*}{D^4}$$

$$\frac{B^e \cdot (C^h)^*}{D^4} = \left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2} D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right]\left[\frac{C^e \cdot (C^h)^*}{j\, D^4}\right]$$

$$-\left[\frac{(\alpha_n D)(\beta D)[(K_{c3} D)^2 - (K_{c2} D)^2]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2}\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{(B^e)^* \cdot C^h}{D^4} = \frac{B^e \cdot (C^h)^*}{D^4}$$

$$\frac{B^e \cdot (C^e)^*}{D^4} = -j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2} D)^2}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$-j\left[\frac{(\alpha_n D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\varepsilon_2 D)(\gamma_2'' D)(K_{c3} D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{j\, D^4}\right]$$

$$\frac{(B^e)^* \cdot C^e}{D^4} = -\frac{B^e \cdot (C^e)^*}{D^4}$$

$$\frac{B^h \cdot (C^e)^*}{D^4} = \left[\frac{(\alpha_n D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega\mu D)(\gamma_2'' D)(K_{c3} D)^2}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2'' D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{jD^4}\right]$$

$$\frac{(B^h)^* \cdot C^e}{D^4} = \frac{B^h \cdot (C^e)^*}{D^4}$$

$$\frac{B^h \cdot (C^h)^*}{D^4} = j\left[\frac{(\omega\epsilon_n D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\mu D)(\gamma_2'' D)(K_{c3}D)^2}\right]\left[\frac{C^e \cdot (C^h)^*}{jD^4}\right]$$

$$-j\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2'' D)(K_{c3}D)^2}\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{(B^h)^* \cdot C^h}{D^4} = \frac{-B^h \cdot (C^h)^*}{D^4}$$

$$\frac{D^e \cdot (D^e)^*}{D^4} = \left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2\left[\frac{1 - \tanh^2[(\gamma_3 D)(h_2/D)]}{\tanh^2[(\gamma_3 D)(h_2/D)]}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$\frac{D^h \cdot (D^h)^*}{D^4} = \left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2\left[1 - \tanh^2[(\gamma_3 D)(h_2/D)]\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{D^e \cdot (D^h)^*}{D^4} = j\left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2\left[\frac{1 - \tanh^2[(\gamma_3 D)(h_2/D)]}{\tanh[(\gamma_3 D)(h_2/D)]}\right]\left[\frac{C^e \cdot (C^h)^*}{jD^4}\right]$$

$$\frac{(D^e)^* \cdot D^h}{D4} = - \frac{D^e \cdot (D^h)^*}{D4}$$

If $(\gamma_2 D)^2 > 0$ and $(\gamma_3 D)^2 < 0$

$$\frac{B^e \cdot (B^e)^*}{D4} = \left[\frac{(\omega\varepsilon_3 D)(\gamma_3'' D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tan\left[(\gamma_3'' D)(h_2/D)\right]}\right]^2 \left[\frac{C^e \cdot (C^e)^*}{D4}\right]$$

$$+2\left[\frac{(\omega\varepsilon_3 D)(\gamma_3'' D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tan\left[(\gamma_3'' D)(h_2/D)\right]}\right] \cdot$$

$$\left[\frac{(\alpha_m D)(\beta D)\left[(K_{c2}D)^2 - (K_{c3}D)^2\right]}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{D4}\right]$$

$$+\left[\frac{(\alpha_m D)(\beta D)\left[(K_{c2}D)^2 - (K_{c3}D)^2\right]}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]^2\left[\frac{C^h \cdot (C^h)^*}{D4}\right]$$

$$\frac{B^h \cdot (B^h)^*}{D4} = \left[\frac{(\alpha_m D)(\beta D)\left[(K_{c2}D)^2 - (K_{c3}D)^2\right]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)}\right]^2\left[\frac{C^e \cdot (C^e)^*}{D4}\right]$$

$$+2\left[\frac{(\alpha_m D)(\beta D)\left[(K_{c2}D)^2 - (K_{c3}D)^2\right]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)}\right] \cdot$$

$$\left[\frac{(\gamma_3''D)(K_{c2}D)^2\tan\left[(\gamma_3''D)(h_2/D)\right]}{(\gamma_2D)(K_{c3}D)^2}\right]\left[\frac{C^e\cdot(C^h)^*}{D^4}\right]$$

$$+\left[\frac{(\gamma_3''D)(K_{c2}D)^2\tan\left[(\gamma_3''D)(h_2/D)\right]}{(\gamma_2D)(K_{c3}D)^2}\right]^2\left[\frac{C^h\cdot(C^h)^*}{D^4}\right]$$

$$\frac{B^e\cdot(B^h)^*}{D^4}=j\left[\frac{-(\omega\varepsilon_3D)(\gamma_3''D)(K_{c2}D)^2}{(\omega\varepsilon_2D)(\gamma_3D)(K_{c3}D)^2\tan\left[(\gamma_3''D)(h_2/D)\right]}\right]\cdot$$

$$\left[\frac{(\alpha_mD)(\beta D)\left[(K_{c2}D)^2-(K_{c3}D)^2\right]}{(\omega\mu D)(\gamma_2D)(K_{c3}D)^2}\right]\left[\frac{C^e\cdot(C^e)^*}{D^4}\right]$$

$$+j\left[\frac{(\omega\varepsilon_3D)(\gamma_3''D)^2(K_{c2}D)^4}{(\omega\varepsilon_2D)(\gamma_2D)^2(K_{c3}D)^4}\right]\left[\frac{(C^e)^*\cdot C^h}{jD^4}\right]$$

$$-j\left[\frac{(\alpha_mD)^2(\beta D)^2\left[(K_{c2}D)^2-(K_{c3}D)^2\right]^2}{(\omega\varepsilon_2D)(\omega\mu D)(\gamma_2D)^2(K_{c3}D)^4}\right]\left[\frac{(C^e)^*\cdot C^h}{D^4}\right]$$

$$+j\left[\frac{(\alpha_mD)(\beta D)\left[(K_{c2}D)^2-(K_{c3}D)^2\right]}{(\omega\varepsilon_2D)(\gamma_2D)(K_{c3}D)^2}\right]\cdot$$

$$\left[\frac{(\gamma_3''D)(K_{c2}D)^2\tan\left[(\gamma_3''D)(h_2/D)\right]}{(\gamma_2D)(K_{c3}D)^2}\right]\left[\frac{C^h\cdot(C^h)^*}{D^4}\right]$$

$$\frac{(B^e)^* \cdot B^h}{D^4} = - \frac{B^e \cdot (B^h)^*}{D^4}$$

$$\frac{B^e \cdot (C^h)^*}{D^4} = j\left[\frac{(\omega \varepsilon_3 D)(\gamma_3'' D)(K_{c2} D)^2}{(\omega \varepsilon_2 D)(\gamma_2 D)(K_{c3} D)^2 \tan[(\gamma_3'' D)(h_2/D)]}\right]\left[\frac{C^e \cdot (C^h)^*}{D^4}\right]$$

$$-j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega \varepsilon_2 D)(\gamma_2 D)(K_{c3} D)^2}\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{(B^e)^* \cdot C^h}{D^4} = - \frac{B^e \cdot (C^h)^*}{D^4}$$

$$\frac{B^e \cdot (C^e)^*}{D^4} = \left[\frac{(\omega \varepsilon_3 D)(\gamma_3'' D)(K_{c2} D)^2}{(\omega \varepsilon_2 D)(\gamma_2 D)(K_{c3} D)^2 \tan[(\gamma_3'' D)(h_2/D)]}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+\left[\frac{(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega \varepsilon_2 D)(\gamma_2 D)(K_{c3} D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{j D^4}\right]$$

$$\frac{(B^e)^* \cdot C^e}{D^4} = \frac{B^e \cdot (C^e)^*}{D^4}$$

$$\frac{B^h \cdot (C^e)^*}{D^4} = j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2} D)^2 - (K_{c3} D)^2]}{(\omega \mu D)(\gamma_2 D)(K_{c3} D)^2}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+j\left[\frac{(\gamma_3''D)(K_{c2}D)^2\tan[(\gamma_3''D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{C^e\cdot(C^h)^*}{D^4}\right]$$

$$\frac{(B^h)^*\cdot C^e}{D^4}=\frac{-B^h\cdot(C^e)^*}{D^4}$$

$$\frac{B^h\cdot(C^h)^*}{D^4}=\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^*\cdot C^h}{jD^4}\right]$$

$$-\left[\frac{(\gamma_3''D)(K_{c2}D)^2\tan[(\gamma_3''D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{C^h\cdot(C^h)^*}{D^4}\right]$$

$$\frac{(B^h)^*\cdot C^h}{D^4}=\frac{B^h\cdot(C^h)^*}{D^4}$$

$$\frac{D^e\cdot(D^e)^*}{D^4}=\left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]\left[\frac{1+\tan^2[(\gamma_3''D)(h_2/D)]}{\tan^2[(\gamma_3''D)(h_2/D)]}\right]\left[\frac{C^e\cdot(C^e)^*}{D^4}\right]$$

$$\frac{D^h\cdot(D^h)^*}{D^4}=\left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]\left[1+\tan^2[(\gamma_3''D)(h_2/D)]\right]\left[\frac{C^h\cdot(C^h)^*}{D^4}\right]$$

$$\frac{D^e\cdot(D^h)^*}{D^4}=\left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]\left[\frac{1+\tan^2[(\gamma_3''D)(h_2/D)]}{\tan[(\gamma_3''D)(h_2/D)]}\right]\left[\frac{C^e\cdot(C^h)^*}{jD^4}\right]$$

$$\frac{(D^e)^* \cdot D^h}{D^4} = \frac{D^e \cdot (D^h)^*}{D^4}$$

If $(\gamma_2 D)^2 > 0$ and $(\gamma_3 D)^2 > 0$

$$\frac{B^e \cdot (B^e)^*}{D^4} = \left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tanh[(\gamma_3 D)(h_3/D)]}\right]^2 \left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+ 2\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tanh[(\gamma_3 D)(h_3/D)]}\right] \cdot$$

$$\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^* C^h}{j D^4}\right]$$

$$+ \left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{B^h \cdot (B^h)^*}{D^4} = \left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)^2}\right]^2 \left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+ 2\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)^2}\right] \cdot$$

$$\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^*\cdot C^h}{D^4}\right]$$

$$+\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]^2\left[\frac{C^h\cdot (C^h)^*}{D^4}\right]$$

$$\frac{B^e\cdot(B^h)^*}{D^4} = -j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)^2(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}\right]\cdot$$

$$\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{C^e\cdot (C^e)^*}{D^4}\right]$$

$$+j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)^2(K_{c2}D)^4}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^4}\right]\left[\frac{C^e\cdot (C^h)^*}{j D^4}\right]$$

$$+j\left[\frac{(\alpha_m D)^2(\beta D)^2[(K_{c2}D)^2-(K_{c3}D)^2]^2}{(\omega\varepsilon_2 D)(\omega\mu D)(\gamma_2 D)^2(K_{c3}D)^4}\right]\left[\frac{C^e\cdot (C^h)^*}{j D^4}\right]$$

$$-j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2-(K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]\cdot$$

$$\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{C^h\cdot (C^h)^*}{D^4}\right]$$

$$\frac{(B^e)^* \cdot B^h}{D^4} = \frac{-B^e \cdot (B^h)^*}{D^4}$$

$$\frac{B^e \cdot (C^h)^*}{D^4} = j\left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tanh[(\gamma_3 D)(h_3/D)]}\right]\left[\frac{C^e \cdot (C^h)^*}{D^4}\right]$$

$$-j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{C^h \cdot (C^h)^*}{D^4}\right]$$

$$\frac{(B^e)^* \cdot C^h}{D^4} = -\frac{B^e \cdot (C^h)^*}{D^4}$$

$$\frac{B^e \cdot (C^e)^*}{D^4} = \left[\frac{(\omega\varepsilon_3 D)(\gamma_3 D)(K_{c2}D)^2}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2 \tanh[(\gamma_3 D)(h_3/D)]}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\varepsilon_2 D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{(C^e)^* \cdot C^h}{jD^4}\right]$$

$$\frac{(B^e)^* \cdot C^e}{D^4} = \frac{B^e \cdot (C^e)^*}{D^4}$$

$$\frac{B^h \cdot (C^e)^*}{D^4} = j\left[\frac{(\alpha_m D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{C^e \cdot (C^e)^*}{D^4}\right]$$

$$+ j\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{(c^e)^* \cdot c^h}{jD^4}\right]$$

$$\frac{(B^h)^* \cdot c^e}{D^4} = -\frac{B^h \cdot (c^e)^*}{D^4}$$

$$\frac{B^h \cdot (c^h)^*}{D^4} = \left[\frac{(\alpha_n D)(\beta D)[(K_{c2}D)^2 - (K_{c3}D)^2]}{(\omega\mu D)(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{(c^e)^* \cdot c^h}{jD^4}\right]$$

$$+\left[\frac{(\gamma_3 D)(K_{c2}D)^2 \tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_2 D)(K_{c3}D)^2}\right]\left[\frac{c^h \cdot (c^h)^*}{D^4}\right]$$

$$\frac{(B^h)^* \cdot c^h}{D^4} = \frac{B^h \cdot (c^h)^*}{D^4}$$

$$\frac{D^e \cdot (D^e)^*}{D^4} = \left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2 \left[\frac{1 - \tanh^2[(\gamma_3 D)(h_2/D)]}{\tanh^2[(\gamma_3 D)(h_2/D)]}\right]\left[\frac{c^e \cdot (c^e)^*}{D^4}\right]$$

$$\frac{D^h \cdot (D^h)^*}{D^4} = \left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2 \left[1 - \tanh^2[(\gamma_3 D)(h_2/D)]\right]\left[\frac{c^h \cdot (c^h)^*}{D^4}\right]$$

$$\frac{D^e \cdot (D^h)^*}{D^4} = j\left[\frac{(K_{c2}D)^2}{(K_{c3}D)^2}\right]^2 \left[\frac{1 - \tanh^2[(\gamma_3 D)(h_2/D)]}{\tanh[(\gamma_3 D)(h_2/D)]}\right]\left[\frac{c^e \cdot (c^h)^*}{jD^4}\right]$$

$$\frac{(D^e)^* \cdot D^h}{D^4} = - \frac{D^e \cdot (D^h)^*}{D^4}$$

REGION 1

The term P_{1a} will be used for the power flow in region 1 for the case $(\gamma_1 D)^2 > 0$.

$$P_{1a} = -\frac{1}{8}\left(\frac{D}{b}\right) \mathrm{Re} \sum_{n=-\infty}^{\infty} \Bigg\{ \Bigg[(\beta D)(\omega \varepsilon_1 D)(\alpha_n D)^2 \frac{A^e \cdot (A^e)^*}{D^4} + (\beta D)(\omega \mu_1 D)$$

$$(\gamma_1 D)^2 \frac{A^h \cdot (A^h)^*}{D^4} - j(\beta D)^2 (\alpha_n D)(\gamma_1 D) \frac{A^e \cdot (A^h)^*}{D^4} + (K_1 D)^2 (\alpha_n D)(\gamma_1 D)$$

$$\frac{(A^e)^* \cdot A^h}{D^4} \Bigg] \Bigg[\frac{2(\gamma_1 D) \tanh[(\gamma_1 D)(h_1/D)]}{(\gamma_1 D)^2 [1 - \tanh^2[(\gamma_1 D)(h_1/D)]]} - 2\left(\frac{h_1}{D}\right) \Bigg] + \Bigg[(\beta D)$$

$$(\omega \mu_1 D)(\alpha_n D)^2 \frac{A^h \cdot (A^h)^*}{D^4} + (\beta D)(\omega \varepsilon_1 D)(\gamma_1 D)^2 \frac{A^e \cdot (A^e)^*}{D^4} -$$

$$j(\beta D)^2 (\alpha_n D)(\gamma_1 D) \frac{A^e \cdot (A^h)^*}{D^4} + j(K_1 D)^2 (\alpha_n D)(\gamma_1 D) \frac{A^h \cdot (A^e)^*}{D^4} \Bigg]$$

$$\Bigg[\frac{2(\gamma_1 D) \tanh[(\gamma_1 D)(h_1/D)]}{(\gamma_1 D)^2 [1 - \tanh^2[(\gamma_1 D)(h_1/D)]]} + 2\left(\frac{h_1}{D}\right) \Bigg] \Bigg\}$$

For the case $(\gamma_1 D)^2 < 0$, γ_1 is imaginary in which case $(\gamma_1'' D)^2 = -(\gamma_1 D)^2$. The power flow, P_{1b}, for this case is

$$P_{1b} = -\frac{1}{8}\left(\frac{D}{b}\right) \mathrm{Re} \sum_{m=-\infty}^{\infty} \Bigg\{ \Bigg[\frac{(\beta D)(\omega \varepsilon_1 D)(\alpha_m D)^2}{(\gamma_1'' D)} \cdot \frac{A^e \cdot (A^e)^*}{D^4} + (\beta D)(\omega \mu_1 D)$$

$$(\gamma_1'' D) \frac{A^h \cdot (A^h)^*}{D^4} - (\beta D)^2 (\alpha_m D) \frac{A^e \cdot (A^h)^*}{D^4} - (K_1 D)^2 (\alpha_m D) \frac{(A^e)^* \cdot A^h}{D^4} \Bigg]$$

$$\left[2(\gamma_1'' D)\left(\frac{h_1}{D}\right) - \sin\left[2(\gamma_1'' D)\left(\frac{h_1}{D}\right)\right]\right] + \Bigg[\frac{(\beta D)(\omega \mu_1 D)(\alpha_m D)^2}{(\gamma_1'' D)} \cdot \frac{A^h \cdot (A^h)^*}{D^4}$$

$$+ (\beta D)(\omega \varepsilon_1 D)(\gamma_1'' D) \frac{A^e \cdot (A^e)^*}{D^4} + (\beta D)^2 (\alpha_m D) \frac{A^e \cdot (A^h)^*}{D^4} + (K_1 D)^2$$

$$(\alpha_m D) \frac{(A^e)^* \cdot A^h}{D^4} \Bigg] \Bigg[2(\gamma_1'' D)(h_1/D) + \sin\left[2(\gamma_1'' D)(h_1/D)\right] \Bigg] \Bigg\}.$$

REGION 2

For region 2, the power flow expressions are the same as for unshielded slotline except that the Fourier integral is replaced by a summation and the interval 2 is replaced by the interval b.

For the case $(\gamma_2 D)^2 > 0$, γ_2 is real and

$$P_{2a} = -\frac{1}{8}\left(\frac{D}{b}\right) \mathrm{Re} \sum_{n=-\infty}^{\infty} \Big\{ \Big[(\beta D)(\omega\varepsilon_2 D)(\alpha_n D) \frac{B^e \cdot (B^e)^*}{D^4} + (\beta D)(\omega\mu_2 D)$$

$$(\alpha_n D)^2 \frac{B^h \cdot (B^h)^*}{D^4} + (\beta D)(\omega\varepsilon_2 D)(\gamma_2 D)^2 \frac{C^e \cdot (C^e)^*}{D^4} + (\beta D)(\omega\mu_2 D)$$

$$(\gamma_2 D)^2 \frac{C^h \cdot (C^h)^*}{D^4} + j\,(\beta D)(\alpha_n D)(\gamma_2 D)\left[\frac{B^e \cdot (C^h)^*}{D^4} + \frac{(B^h)^* \cdot C^e}{D^4}\right] -$$

$$j\,(K_2 D)^2 (\alpha_n D)(\gamma_2 D)\left[\frac{(B^e)^* \cdot C^h}{D^4} + \frac{B^h \cdot (C^e)^*}{D^4}\right]\Big]\left[\frac{2(\gamma_2 D)\tanh(\gamma_2 D)}{(\gamma_2 D)[1-\tanh^2(\gamma_2 D)]}\right.$$

$$\left. -2\right] + \Big[(\beta D)(\omega\varepsilon_2 D)(\alpha_n D)^2 \frac{C^e \cdot (C^e)^*}{D^4} + (\beta D)(\omega\mu_2 D)(\alpha_n D)^2 \frac{B^h \cdot (B^h)^*}{D^4}$$

$$+ j\,(\beta D)^2 (\alpha_n D)(\gamma_2 D)\left[\frac{(B^h)^* \cdot C^e}{D^4} + \frac{B^e \cdot (C^h)^*}{D^4}\right] - j\,(K_2 D)^2 (\alpha_n D)(\gamma_2 D)$$

$$\left[\frac{(B^h)^* \cdot C^e}{D^4} + \frac{(B^e)^* \cdot C^h}{D^4}\right]\left[\frac{2(\gamma_2 D)\tanh(\gamma_2 D)}{(\gamma_2 D)^2[1-\tanh^2(\gamma_2 D)]} + 2\right] +$$

$$\Big[(\beta D)(\omega\varepsilon_2 D)\left[(\alpha_n D)^2 + (\gamma_2 D)^2\right]\left[\frac{B^e \cdot (C^e)^*}{D^4} + \frac{(B^e)^* \cdot C^e}{D^4}\right] +$$

$$(\beta D)(\omega\mu_2 D)\left[(\alpha_n D)^2 + (\gamma_2 D)^2\right]\left[\frac{B^h\cdot(C^h)^*}{D^4} + \frac{(B^h)^*\cdot C^h}{D^4}\right]$$

$$+j2(\beta D)^2(\alpha_n D)(\gamma_2 D)\left[\frac{B^e\cdot(B^h)^*}{D^4} + \frac{C^e\cdot(C^h)^*}{D^4}\right] - j2(K_2 D)^2$$

$$(\alpha_n D)(\gamma_2 D)\left[\frac{(C^e)^*\cdot C^h}{D^4} + \frac{(B^e)^*\cdot B^h}{D^4}\right]\Bigg]\left[\frac{2\tanh^2(\gamma_2 D)}{(\gamma_2 D)[1-\tanh^2(\gamma_2 D)]}\right]\Bigg\}.$$

For the case $(\gamma_2 D)^2 < 0$, γ_2 is imaginary in which case $(\gamma_2'' D)^2 = -(\gamma_2 D)^2$. The power flow, P_{2b}, for this case is

$$P_{2b} = -\frac{1}{8}\left(\frac{D}{b}\right)\mathrm{Re}\sum_{n=-\infty}^{\infty}\Bigg\{\Bigg[\frac{(\beta D)(\omega\varepsilon_2 D)(\alpha_n D)^2}{(\gamma_2'' D)}\,\frac{B^e\cdot(B^e)^*}{D^4} + \frac{(\beta D)}{(\gamma_2'' D)}$$

$$\frac{(\alpha_n D)^2 B^h\cdot(B^h)^*}{D^4} + (\beta D)(\omega\varepsilon_2 D)(\gamma_2'' D)\frac{C^e\cdot(C^e)^*}{D^4} + (\beta D)(\omega\mu_2 D)$$

$$(\gamma_2'' D)\,\frac{C^h\cdot(C^h)^*}{D^4} + (\beta D)^2(\alpha_n D)\left[\frac{B^e\cdot(C^h)^*}{D^4} - \frac{(B^h)^*\cdot C^e}{D^4}\right] +$$

$$(K_2 D)^2(\alpha_n D)\left[\frac{(B^e)^*\cdot C^h}{D^4} - \frac{B^h\cdot(C^e)^*}{D^4}\right]\Bigg]\left[2\gamma_2'' D - \sin(2\gamma_2'' D)\right] +$$

$$\left[\frac{(\beta D)(\omega\varepsilon_2 D)(\alpha_m D)^2}{(\gamma_2'' D)} \cdot \frac{C^e \cdot (C^e)^*}{D^4} + \frac{(\beta D)(\omega\mu_2 D)(\alpha_m D)^2}{(\gamma_2'' D)} \cdot \frac{C^h \cdot (C^h)^*}{D^4} + \right.$$

$$(\beta D)(\omega\varepsilon_2 D)(\gamma_2'' D)\frac{B^e \cdot (B^e)^*}{D^4} + (\beta D)(\omega\mu_2 D)(\gamma_2'' D)\frac{B^h \cdot (B^h)^*}{D^4} +$$

$$\left.(\beta D)^2(\alpha_m D)\left[\frac{(B^h)^* \cdot C^e}{D^4} - \frac{B^e (C^h)^*}{D^4}\right] + (K_2 D)^2(\alpha_m D)\left[\frac{B^h \cdot (C^e)^*}{D^4} - \frac{(B^e)^* \cdot C^h}{D^4}\right]\right]$$

$$\left[2(\gamma_2'' D) + \sin(2\gamma_2'' D)\right] + j\left[(\beta D)(\omega\varepsilon_2 D)\left[\frac{(\alpha_m D)^2 - (\gamma_2'' D)^2}{(\gamma_2'' D)}\right]\right.$$

$$\left[\frac{B^e \cdot (C^e)^*}{D^4} - \frac{(B^e)^* \cdot C^e}{D^4}\right] + (\beta D)(\omega\mu_2 D)\left[\frac{(\alpha_m D)^2 - (\gamma_2'' D)^2}{(\gamma_2'' D)}\right]$$

$$\left[\frac{B^h \cdot (C^h)^*}{D^4} - \frac{(B^h)^* \cdot C^h}{D^4}\right] + 2(\beta D)^2(\alpha_m D)\left[\frac{B^e \cdot (B^h)^*}{D^4} - \frac{C^e \cdot (C^h)^*}{D^4}\right] +$$

$$\left.2(K_2 D)^2(\alpha_m D)\left[\frac{(C^e)^* \cdot C^h}{D^4} - \frac{(B^e)^* \cdot B^h}{D^4}\right]\left[1 - \cos(2\gamma_2'' D)\right]\right\}.$$

REGION 3

For the case $(\gamma_3 D)^2 > 0$, is real and the power flow, P_{3a}, is

$$P_{3a} = -\frac{1}{8}\left(\frac{D}{b}\right) \mathrm{Re} \sum_{n=-\infty}^{\infty} \left\{ \left[(\beta D)(\omega\varepsilon_3 D)(\alpha_n D)^2 \frac{D^e \cdot (D^e)^*}{D^4} + (\beta D)(\omega\mu_3 D) \right.\right.$$

$$(\gamma_3 D)^2 \frac{D^h \cdot (D^h)^*}{D^4} + j\,(\beta D)^2(\alpha_n D)(\gamma_3 D) \frac{D^e \cdot (D^h)^*}{D^4} - j\,(K_3 D)^2(\alpha_n D)$$

$$\left.(\gamma_3 D) \frac{(D^e)^* \cdot D^h}{D^4}\right]\left[\frac{2(\gamma_3 D)\tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_3 D)[1-\tanh^2[(\gamma_3 D)(h_2/D)]]} - 2\left(\frac{h_2}{D}\right)\right] +$$

$$\left[(\beta D)(\omega\mu_3 D)(\alpha_n D)^2 \frac{D^h \cdot (D^h)^*}{D^4} + (\beta D)(\omega\varepsilon_3 D)(\gamma_3 D)^2 \frac{D^e \cdot (D^e)^*}{D^4} + \right.$$

$$\left. j\,(\beta D)^2(\alpha_n D)(\gamma_3 D) \frac{D^e \cdot (D^h)^*}{D^4} - j\,(K_3 D)^2(\alpha_n D)(\gamma_3 D) \frac{(D^e)^* \cdot D^h}{D^4} \right]$$

$$\left.\left[\frac{2(\gamma_3 D)\tanh[(\gamma_3 D)(h_2/D)]}{(\gamma_3 D)^2[1-\tanh^2[(\gamma_3 D)(h_2/D)]]} + 2\left(\frac{h_2}{D}\right)\right]\right\}.$$

For the case $(\gamma_3 D)^2 < 0$, γ_3 is imaginary in which case $(\gamma_3'' D)^2 = -(\gamma_3 D)^2$. The power flow, P_{3b}, for this case is

$$P_{3b} = -\frac{1}{8}\left(\frac{D}{b}\right) \mathrm{Re} \sum_{n=-\infty}^{\infty} \left\{ \left[\frac{(\beta D)(\omega\varepsilon_2 D)(\alpha_n D)^2}{(\gamma_3'' D)} \frac{D^e \cdot (D^e)^*}{D^4} + (\beta D)(\omega\mu_3 D) \right.\right.$$

$$(\gamma_3''D)\,\frac{D^h\cdot(D^h)^*}{D^4}+(\beta D)^2(\alpha_m D)\,\frac{D^e\cdot(D^h)^*}{D^4}+(K_3D)^2(\alpha_m D)\,\frac{(D^e)^*\cdot D^h}{D^4}\Bigg]$$

$$\left[2(\gamma_3''D)\left(\frac{h_2}{D}\right)-\sin\left[2(\gamma_3''D)\left(\frac{h_2}{D}\right)\right]\right]+\Bigg[\frac{(\beta D)(\omega\mu_3 D)(\alpha_m D)^2}{(\gamma_3''D)}$$

$$\frac{D^h\cdot(D^h)^*}{D^4}+(\beta D)(\omega\varepsilon_3 D)(\gamma_3''D)\,\frac{D^e\cdot(D^e)^*}{D^4}-(\beta D)^2(\alpha_m D)\,\frac{D^e\cdot(D^h)^*}{D^4}$$

$$-(K_3D)^2(\alpha_m D)\,\frac{(D^e)^*\cdot D^h}{D^4}\Bigg]\left[2(\gamma_3''D)\left(\frac{h_2}{D}\right)+\sin\left[2(\gamma_3''D)\left(\frac{h_2}{D}\right)\right]\right]\Bigg\}.$$

APPENDIX C

COMPUTER PROGRAM 'FINIMP'

```
C***********************************************************************
C         COMPUTATION OF FIN-LINE IMPEDANCE                            *
C         PROGRAMMER: MAJOR KIM, BYUNGYONG ,KOREAN AIRFORCE            *
C         SUPERVISED BY DR. J.B. KNORR                                 *
C         LANGUAGE: FORTRAN BY USING IBM 3033                          *
C         U.S. NPGS,MONTEREY, CALIFORNIA                               *
C         AUGUST 1984                                                  *
C                                                                      *
C****************INPUT DATA FILE***************************************
C                                                                      *
C D/LAMBDA             NORMALIZED FREQUENCIES                          *
C EPSR1,2,3            RELATIVE DIELECTRIC CONSTANTS FOR REGIONS 1,2,3 *
C H1/D, H2/D, B/D      NORMALIZED FIN POSITION COORDS AND WG HEIGTH    *
C W/B                  NORMALIZED FIN GAP WIDTHS                       *
C                                                                      *
C     HERE W = FIN GAP WIDTH                                           *
C          B = HEIGTH OF RECTANGULAR WAVEGUIDE                         *
C          D = DIELECTRIC THICKNESS                                    *
C          LAMBDA = FREE SPACE WAVELENGTH                              *
C                                                                      *
C*****VARIABLE DEFINITIONS*********************************************
C   ALFD = NORMALIZED 'X' TRANSFORM VARIABLE                           *
C   BETAD = NORMALIZED 'Z' TRANSFORM VARIABLE                          *
C   BOVD = WAVEGUIDE HEIGTH/D                                          *
C   BOVW = WAVEGUIDE HEIGTH/FIN GAP WIDTH                              *
C   DOVL = D/FREE SPACE WAVELENGTH                                     *
C   EPSR1,2,3 = RELATIVE DIELECTRIC CONSTANTS FO REGIONS 1,2,3 RESP.   *
C   H1OVD = FIN POSITION COORDINATE/D                                  *
C   H2OVD = FIN POSITION COORDINATE/D                                  *
C   LPOVL = GUIDE WAVELENGTH/FREE SPACE WAVELENGTH                     *
C   WOVB = INVERSE OF BOVW                                             *
C   XCONST = CONSTANT ADJUSTING THE LIMIT OF SUMMATION OVER ALFD       *
C   IMP = FIN-LINE CHARACTERISTIC IMPEDANCE                            *
C                                                                      *
C*****VARIABLE DECLARATION*********************************************
      REAL LPOVL,KC1DSQ,KC2DSQ,KC3DSQ,IMP,PWR1,PWR2,PWR3,K1DSQ,K2DSQ,
     1     K3DSQ
      COMPLEX C,AEAHC,AECAH,CECHC,CECCH,BHBHC,BEBHC,BECBH,BECHC,BECCH,
     1        BHCEC,BHCCE,BECEC,BECCE,BHCHC,BHCCH,DEDHC,DECDH,
     1        AEAHCP,AECAHP,CECHCP,CECCHP,BHBHCP,BEBHCP,BECBHP,BECHCP,
     1        BECCHP,BHCECP,BHCCEP,BECECP,BECCEP,BHCHCP,BHCCHP,
     1        DEDHCP,DECDHP,P11,P12,P1,P21,P22,P23,P2,P31,P32,P3,PWR
      DIMENSION WOVBI(6)
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      COMMON/C5/XCONST
C   SPECIFY LIMIT OF SUMMATION CONSTANTS FOR THE X FIELDS
```

```
      XCONST = 22.0
C   READ INPUT DATA
      EPSR1=1.
      EPSR2=2.2
      EPSR3=1.
      H1OVD=18.8
      H2OVD=17.8
      BOVD=18.8
      WOVBI(1)=1.0
      WOVBI(2)=0.5
      WOVBI(3)=0.2
      WOVBI(4)=0.1
      WOVBI(5)=0.05
      WOVBI(6)=0.02
C   DEFINE CONSTANTS
      PI = ARCOS(-1.0)
      C2PI=6.283185307
      C2PISQ=C2PI**2
C   SPECIFY FIN GEOMETRY
      DO 12 IW=1,6
         WOVB=WOVBI(IW)
         WOVD=WOVB*BOVD
         IF(WOVB.EQ.0.) GO TO 13
C   SPECIFY FREQUENCY (DOVL)
         DO 11 ID=40,60
            FREQ=FLOAT(ID)
            DOVL=0.0127*FREQ/30.
            IF(DOVL.EQ.0.) GO TO 12
C SEARCH FOR ZERO OF IPROD1
C REF: GOTTFRIED,PROGRAMMING WITH FORTRAN IV, PG. 157
C SEARCH INTERVAL IS XL=.1 TO XR=3
            XL=.1
            IF(FREQ.GE.53) XR=1.6
            IF(FREQ.LT.53) XR=3.
            EPSLN=.0013579
            ITER=1
C   CALCULATE INTERIOR POINTS
    1       XL1=XL+.5*(XR-XL-EPSLN)
            XR1=XL1+EPSLN
C   SUBROUTINE IPROD USES COMMON BLOCKS /C1/C2/C3/
            CALL IPROD1(XL1,YL1)
            CALL IPROD1(XR1,YR1)
            IF(YL1**2-YR1**2) 2,5,3
C   YR1**2 GREATER THAN YL1**2
    2       XR=XR1
            GO TO 4
C   YL1**2 GREATER THAN YR1**2
    3       XL=XL1
```

```
C   TEST FOR END OF SEARCH
    4       IF(ITER.GE.100) GO TO 6
            ITER=ITER+1
            IF(XR-XL.GT.3.*EPSLN) GO TO 1
    5       LPOVL=.5*(XL1+XR1)
            GO TO 8
C   WRITE OUTPUT - SEARCH FAILED TO CONVERGE
    6       WRITE(6,610)
            GO TO 11
    8     CONTINUE
C CALCULATE THE CHARACTERISTIC IMPEDANCE
          BETAD = C2PI*DOVL/LPOVL
          BOVW = 1./WOVB
          PWR=CMPLX(0.0,0.0)
          PWR1=0.0
          PWR2=0.0
          PWR3=0.0
          M=50
          IF(WOVD.EQ.BOVD) M=1
       DO 14 L=1,M
          N=L-1
          ALFD=FLOAT(N)*C2PI/BOVD
          IF(ALFD.EQ.0.) EX=1.
          IF(ALFD.GT.0.) EX=SIN(.5*ALFD*WOVD)/(.5*ALFD*WOVD)
          EZ=0.0
C   CALCULATE VARIABLES DEPENDENT ON FREQUENCY
          WMUD=60.*C2PISQ*DOVL
          WEPS1D=EPSR1*DOVL/60.
          WEPS2D=EPSR2*DOVL/60.
          WEPS3D=EPSR3*DOVL/60.
C   CALCULATE VARIABLES DEPENDENT ON FRRQUENCY AND BETAD
          BETDSQ=BETAD**2
          KC1DSQ=C2PISQ*EPSR1*DOVL**2-BETDSQ
          KC2DSQ=C2PISQ*EPSR2*DOVL**2-BETDSQ
          KC3DSQ=C2PISQ*EPSR3*DOVL**2-BETDSQ
          K1DSQ =WMUD*WEPS1D
          K2DSQ =WMUD*WEPS2D
          K3DSQ =WMUD*WEPS3D
C   CALCULATE VARIABLES DEPENDENT ON FREQUENCY, BETAD AND ALFD
          ALFDSQ=ALFD**2
          G1DSQ=ALFDSQ-KC1DSQ
          G2DSQ=ALFDSQ-KC2DSQ
          G3DSQ=ALFDSQ-KC3DSQ
          G1D=SQRT(ABS(G1DSQ))
          G2D=SQRT(ABS(G2DSQ))
          G3D=SQRT(ABS(G3DSQ))
          CALL TFN(G1DSQ,H1OVD,TFN1)
          CALL TFN(G2DSQ,1.,TFN2)
```

```
      CALL TFN(G3DSQ,H2OVD,TFN3)
      D11=-KC2DSQ*(1.+(WEPS3D*G3DSQ*KC2DSQ*TFN2)/
   1      (WEPS2D*G2DSQ*KC3DSQ*TFN3))
      D12=((ALFD*BETAD)/(WEPS2D*G2DSQ))*((KC2DSQ/KC1DSQ)-1.)*KC2DSQ
      D21=-ALFD*BETAD*((KC2DSQ/KC1DSQ)+((WEPS3D*KC2DSQ*G3DSQ*TFN2)/
   1      (WEPS2D*KC3DSQ*G2DSQ*TFN3)))
      D22=WMUD*(1.+((KC2DSQ*TFN3)/(KC3DSQ*TFN2))+((ALFDSQ*BETDSQ)/
   1      (G2DSQ*WMUD*WEPS2D))*((KC2DSQ/KC3DSQ)-1.))
      DET=D11*D22-D21*D12
      CALL TFNS(G1DSQ,H1OVD,TFN21)
      CALL TFNS(G2DSQ,1.,TFN22)
      CALL TFNS(G3DSQ,H2OVD,TFN23)
      CALL TFNSQ(G1DSQ,H1OVD,TFNSQ1)
      CALL TFNSQ(G2DSQ,1.,TFNSQ2)
      CALL TFNSQ(G3DSQ,H2OVD,TFNSQ3)
      C=CMPLX(0.0,1.0)
C
C
      IF(G1DSQ) 21,22,22
C
C   G1DSQ IS LESS THAN ZERO
 21   AEAEC=((EZ**2)*(1.+TFNSQ1))/((K1DSQ**2)*TFNSQ1)
      AHAHC=((KC1DSQ*EX-ALFD*BETAD*EZ)**2*(1.+TFNSQ1))/
   1        ((WMUD**2)*(G1D**2)*(KC1DSQ**2)*TFNSQ1)
      AEAH =(EZ*(KC1DSQ*EX-ALFD*BETAD*EZ)*(1.+TFNSQ1))/
   1        (WMUD*G1D*(KC1DSQ**2)*TFNSQ1)
      AEAHC=CMPLX(-AEAH,0.0)
      AECAH=AEAHC
C
C
C   POWER IN REGION 1
      P1=(((BETAD*WEPS1D*ALFD**2*AEAEC)/G1D)+(BETAD*WMUD*G1D*AHAHC)
   1     -(BETAD**2*ALFD*AEAHC)-(K1DSQ*ALFD*AECAH))*(2.*G1D*H1OVD-
   1     SIN(2.*G1D*H1OVD))+
   1     (((BETAD*WMUD*ALFD**2*AHAHC)/G1D)+(BETAD*WEPS1D*G1D*AEAEC)
   1     +(BETAD**2*ALFD*AEAHC)+(K1DSQ*ALFD*AECAH))*(2.*G1D*H1OVD+
   1     SIN(2.*G1D*H1OVD))
        GO TO 23
C
C
C   G1DSQ IS GREATER THAN ZERO
 22   AEAEC=((EZ**2)*(1.-TFNSQ1))/((KC1DSQ**2)*TFNSQ1)
      AHAHC=((KC1DSQ*EX-ALFD*BETAD*EZ)**2*(1.-TFNSQ1))/
   1        ((WMUD**2)*G1D**2*(KC1DSQ**2)*TFNSQ1)
      AEAH =(EZ*(KC1DSQ*EX-ALFD*BETAD*EZ)*(1.-TFNSQ1))/
   1        (WMUD*G1D*(KC1DSQ**2)*TFNSQ1)
      AEAHC=CMPLX(0.0,AEAH)
      AECAH=-AEAHC
```

```
      AEAECP=(EZ**2)/((KC1DSQ**2)*TFNSQ1)
      AHAHCP=((KC1DSQ*EX-ALFD*BETAD*EZ)**2)/
     1       ((WMUD**2)*G1D**2*(KC1DSQ**2)*TFNSQ1)
      AEAHP =(EZ*(KC1DSQ*EX-ALFD*BETAD*EZ))/
     1       (WMUD*G1D*(KC1DSQ**2)*TFNSQ1)
      AEAHCP=CMPLX(0.0,AEAHP)
      AECAHP=-AEAHCP
C
C
C    POWER IN REGION 1
      P1A=BETAD*WEPS1D*ALFD**2
      P1B=BETAD*WMUD*G1D**2
      P1C=BETAD**2*ALFD*G1D
      P1D=K1DSQ*ALFD*G1D
      P1E=BETAD*WMUD*ALFD**2
      P1F=BETAD*WEPS1D*G1D**2
      P1G=2.*TFN21/(G1D**2)
      P11=(P1A*AEAEC+P1B*AHAHC-C*P1C*AEAHC+C*P1D*AECAH)*(-2.*H1OVD)+
     1       (P1E*AHAHC+P1F*AEAEC-C*P1C*AEAHC+C*P1D*AECAH)*(2.*H1OVD)
      P12=(P1A*AEAECP+P1B*AHAHCP-C*P1C*AEAHCP+C*P1D*AECAHP)*P1G+
     1       (P1E*AHAHCP+P1F*AEAECP-C*P1C*AEAHCP+C*P1D*AECAHP)*P1G
      P1 =P11+P12
C
C
   23 IF(G2DSQ) 24,25,25
C
C    G2DSQ IS LESS THAN ZERO
   24 CECEC=((D12*EX-D22*EZ)**2*(1.+TFNSQ2))/DET**2
      CHCHC=((D21*EZ-D11*EX)**2*(1.+TFNSQ2))/(DET**2*G2D**2*TFNSQ2)
      CECH =((D12*EX-D22*EZ)*(D21*EZ-D11*EX)*(1.+TFNSQ2))/
     1       (DET**2*TFN22)
      CECHC=CMPLX(0.0,-CECH)
      CECCH=-CECHC
C
C
      IF(G3DSQ) 26,27,27
C
C    G2DSQ IS LESS THAN ZERO AND  G3DSQ IS LESS THAN ZERO
   26 BEBE1=((WEPS3D*G3D**2*KC2DSQ)**2*CECEC)/((WEPS2D*G2D*KC3DSQ*
     1       TFN23)**2)
      BEBE2=(2.*WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC3DSQ-KC2DSQ)*
     1       CECH)/((WEPS2D*G2D*KC3DSQ)**2*TFN23)
      BEBE3=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CHCHC)/
     1       ((WEPS2D*G2D*KC3DSQ)**2)
      BEBEC=BEBE1-BEBE2+BEBE3
      BHBH1=((ALFD*BETAD*(KC2DSQ-KC3DSQ))**2*CECEC)/
     1       ((WMUD*G2D*KC3DSQ)**2)
      BHBH2=(2.*ALFD*BETAD*(KC2DSQ-KC3DSQ)*KC2DSQ*TFN23*CECH)/
```

```
1         (WMUD*G2D**2*KC3DSQ**2)
      BHBH3=((KC2DSQ*TFN23)**2*CHCHC)/((G2D*KC3DSQ)**2)
      BHBHC=BHBH1-BHBH2+BHBH3
      BEBH1=(WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECEC)/
1         (WEPS2D*G2D**2*KC3DSQ**2*TFN23*WMUD)
      BEBH2=(WEPS3D*G3D**2*KC2DSQ**2*CECH)/(WEPS2D*G2D**2*KC3DSQ**2)
      BEBH3=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CECH)/
1         (WEPS2D*G2D**2*KC3DSQ**2*WMUD)
      BEBH4=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*KC2DSQ*TFN23*CHCHC)/
1         (WEPS2D*G2D**2*KC3DSQ**2)
      BEBHC=CMPLX(0.0,-BEBH1)+CMPLX(0.0,BEBH2)+
1         CMPLX(0.0,-BEBH3)+CMPLX(0.0,-BEBH4)
      BECBH=-BEBHC
      BECH1=(WEPS3D*G3D**2*KC2DSQ*CECH)/(WEPS2D*G2D*KC3DSQ*TFN23)
      BECH2=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CHCHC)/(WEPS2D*G2D*KC3DSQ)
      BECHC=CMPLX(-BECH1,0.0)+CMPLX(BECH2,0.0)
      BECCH=BECHC
      BHCE1=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECEC)/(WMUD*G2D*KC3DSQ)
      BHCE2=(KC2DSQ*TFN23*CECH)/(G2D*KC3DSQ)
      BHCEC=CMPLX(BHCE1,0.0)-CMPLX(BHCE2,0.0)
      BHCCE=BHCEC
      BECE1=(WEPS3D*G3D**2*KC2DSQ*CECEC)/(WEPS2D*G2D*KC3DSQ*TFN23)
      BECE2=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CECH)/(WE[illegible]D*G2D*KC3DSQ)
      BECEC=CMPLX(0.0,-BECE1)+CMPLX(0.0,BECE2)
      BECCE=-BECEC
      BHCH1=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECH)/(WMUD*G2D*KC3DSQ)
      BHCH2=(KC2DSQ*TFN23*CHCHC)/(G2D*KC3DSQ)
      BHCHC=CMPLX(0.0,-BHCH1)+CMPLX(0.0,BHCH2)
      BHCCH=-BHCHC
C
C
C     POWER IN REGION 2
      P21=((BETAD*WEPS2D*ALFD**2*BEBEC/G2D)+(BETAD*WMUD*ALFD**2*
1        BHBHC/G2D)+(BETAD*WEPS2D*G2D*CECEC)+(BETAD*WMUD*G2D*
1        CHCHC)+(BETAD**2*ALFD*(BECHC-BHCCE))+(K2DSQ*ALFD*
1        (BECCH-BHCEC)))*(2.*G2D-SIN(2.*G2D))
      P22=((BETAD*WEPS2D*ALFD**2*CECEC/G2D)+(BETAD*WMUD*ALFD**2*
1        CHCHC/G2D)+(BETAD*WEPS2D*G2D*BEBEC)+(BETAD*WMUD*G2D*
1        BHBHC)+(BETAD**2*ALFD*(BHCCE-BECHC))+(K2DSQ*ALFD*
1        (BHCEC-BECCH)))*(2.*G2D+SIN(2.*G2D))
      P23=((BETAD*WEPS2D*(ALFD**2-G2D**2)/G2D*(BECEC-BECCE))+
1        (BETAD*WMUD*(ALFD**2-G2D**2)/G2D*(BHCHC-BHCCH))+
1        (2.*BETAD**2*ALFD*(BEBHC-CECHC))+
1        (2.*K2DSQ*ALFD*(CECCH-BECBH)))*(1.-COS(2.*G2D))
      P2=P21+P22+C*P23
C
C
      DEDEC=(KC2DSQ**2*(1.+TFNSQ3)*CECEC)/(KC3DSQ**2*TFNSQ3)
```

```
      DHDHC=(KC2DSQ**2*(1.+TFNSQ3)*CHCHC)/(KC3DSQ**2)
      DEDH=-(KC2DSQ**2*(1.+TFNSQ3)*CECH*G3D)/(KC3DSQ**2*TFN23)
      DEDHC=CMPLX(DEDH,0.0)
      DECDH=DEDHC
C
C
C    POWER IN REGION 3
      P31=((BETAD*WEPS3D*ALFD**2*DEDEC/G3D)+(BETAD*WMUD*G3D*DHDHC)+
     1    (BETAD**2*ALFD*DEDHC)+(K3DSQ*ALFD*DECDH))*(2.*G3D*H2OVD-
     1    SIN(2.*G3D*H2OVD))
      P32=((BETAD*WMUD*ALFD**2*DHDHC/G3D)+(BETAD*WEPS3D*G3D*DEDEC)-
     1    (BETAD**2*ALFD*DEDHC)-(K3DSQ*ALFD*DECDH))*(2.*G3D*H2OVD+
     1    SIN(2.*G3D*H2OVD))
      P3=P31+P32
      GO TO 30
C
C
C    G2DSQ IS LESS THAN ZERO AND  G3DSQ IS GREATER THAN ZERO
   27 BEBE1=((WEPS3D*G3D**2*KC2DSQ)**2*CECEC)/((WEPS2D*G2D*KC3DSQ*
     1      TFN23)**2)
      BEBE2=(2.*WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC3DSQ-KC2DSQ)*
     1      CECH)/((WEPS2D*G2D*KC3DSQ)**2*TFN23)
      BEBE3=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CHCHC)/
     1      ((WEPS2D*G2D*KC3DSQ)**2)
      BEBEC=BEBE1-BEBE2+BEBE3
      BHBH1=((ALFD*BETAD*(KC2DSQ-KC3DSQ))**2*CECEC)/
     1      ((WMUD*G2D*KC3DSQ)**2)
      BHBH2=(2.*ALFD*BETAD*(KC2DSQ-KC3DSQ)*KC2DSQ*TFN23*CECH)/
     1      (WMUD*G2D**2*KC3DSQ**2)
      BHBH3=((KC2DSQ*TFN23)**2*CHCHC)/((G2D*KC3DSQ)**2)
      BHBHC=BHBH1+BHBH2+BHBH3
      BEBH1=(WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECEC)/
     1      (WEPS2D*G2D**2*KC3DSQ**2*TFN23*WMUD)
      BEBH2=(WEPS3D*G3D**2*KC2DSQ**2*CECH)/(WEPS2D*G2D**2*KC3DSQ**2)
      BEBH3=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CECH)/
     1      (WEPS2D*G2D**2*KC3DSQ**2*WMUD)
      BEBH4=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*KC2DSQ*TFN23*CHCHC)/
     1      (WEPS2D*G2D**2*KC3DSQ**2)
      BEBHC=CMPLX(0.0,-BEBH1)+CMPLX(0.0,-BEBH2)+
     1      CMPLX(0.0,-BEBH3)+CMPLX(0.0,BEBH4)
      BECBH=-BEBHC
      BECH1=(WEPS3D*G3D**2*KC2DSQ*CECH)/(WEPS2D*G2D*KC3DSQ*TFN23)
      BECH2=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CHCHC)/(WEPS2D*G2D*KC3DSQ)
      BECHC=CMPLX(-BECH1,0.0)+CMPLX(BECH2,0.0)
      BECCH=BECHC
      BHCE1=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECEC)/(WMUD*G2D*KC3DSQ)
      BHCE2=(KC2DSQ*TFN23*CECH)/(G2D*KC3DSQ)
      BHCEC=CMPLX(BHCE1,0.0)+CMPLX(BHCE2,0.0)
```

```
      BHCCE=BHCEC
      BECE1=(WEPS3D*G3D**2*KC2DSQ*CECEC)/(WEPS2D*G2D*KC3DSQ*TFN23)
      BECE2=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CECH)/(WEPS2D*G2D*KC3DSQ)
      BECEC=CMPLX(0.0,-BECE1)+CMPLX(0.0,BECE2)
      BECCE=-BECEC
      BHCH1=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECH)/(WMUD*G2D*KC3DSQ)
      BHCH2=(KC2DSQ*TFN23*CHCHC)/(G2D*KC3DSQ)
      BHCHC=CMPLX(0.0,-BHCH1)+CMPLX(0.0,-BHCH2)
      BHCCH=-BHCHC
C
C
C     POWER IN REGION 2
      P21=((BETAD*WEPS2D*ALFD**2*BEBEC/G2D)+(BETAD*WMUD*ALFD**2*
     1      BHBHC/G2D)+(BETAD*WEPS2D*G2D*CECEC)+(BETAD*WMUD*G2D*
     1      CHCHC)+(BETAD**2*ALFD*(BECHC-BHCCE))+(K2DSQ*ALFD*
     1      (BECCH-BHCEC)))*(2.*G2D-SIN(2.*G2D))
      P22=((BETAD*WEPS2D*ALFD**2*CECEC/G2D)+(BETAD*WMUD*ALFD**2*
     1      CHCHC/G2D)+(BETAD*WEPS2D*G2D*BEBEC)+(BETAD*WMUD*G2D*
     1      BHBHC)+(BETAD**2*ALFD*(BHCCE-BECHC))+(K2DSQ*ALFD*
     1      (BHCEC-BECCH)))*(2.*G2D+SIN(2.*G2D))
      P23=((BETAD*WEPS2D*(ALFD**2-G2D**2)/G2D*(BECEC-BECCE))+
     1      (BETAD*WMUD*(ALFD**2-G2D**2)/G2D*(BHCHC-BHCCH))+
     1      (2.*BETAD**2*ALFD*(BEBHC-CECHC))+
     1      (2.*K2DSQ*ALFD*(CECCH-BECBH)))*(1.-COS(2.*G2D))
      P2=P21+P22+C*P23
C
C
      DEDEC=(KC2DSQ**2*(1.-TFNSQ3)*CECEC)/(KC3DSQ**2*TFNSQ3)
      DHDHC=(KC2DSQ**2*(1.-TFNSQ3)*CHCHC)/(KC3DSQ**2)
      DEDH =-(KC2DSQ**2*(1.-TFNSQ3)*CECH*G3D)/(KC3DSQ**2*TFN23)
      DEDHC=CMPLX(0.0,DEDH)
      DECDH=-DEDHC
      DEDECP=(KC2DSQ**2*CECEC)/(KC3DSQ**2*TFNSQ3)
      DHDHCP=(KC2DSQ**2*CHCHC)/(KC3DSQ**2)
      DEDHP =-(KC2DSQ**2*CECH*G3D)/(KC3DSQ**2*TFN23)
      DEDHCP=CMPLX(0.0,DEDHP)
      DECDHP=-DEDHCP
C
C
C     POWER IN REGION 3
      P3A=BETAD*WEPS3D*ALFD**2
      P3B=BETAD*WMUD*G3D**2
      P3C=BETAD**2*ALFD*G3D
      P3D=K3DSQ*ALFD*G3D
      P3E=BETAD*WMUD*ALFD**2
      P3F=BETAD*WEPS3D*G3D**2
      P3G=2.*TFN23/G3D**2
      P31=(P3A*DEDEC+P3B*DHDHC+C*P3C*DEDHC-C*P3D*DECDH)*(-2.*H2OVD)+
```

```
     1     (P3E*DHDHC+P3F*DEDEC+C*P3C*DEDHC-C*P3D*DECDH)*(2.*H2OVD)
      P32=(P3A*DEDECP+P3B*DHDHCP+C*P3C*DEDHCP-C*P3D*DECDHP)*P3G+
     1     (P3E*DHDHCP+P3F*DEDECP+C*P3C*DEDHCP-C*P3D*DECDHP)*P3G
      P3 =P31+P32
      GO TO 30
C
C
C   G2DSQ IS GREATER THAN ZERO
 25   CECEC=((D12*EX-D22*EZ)**2*(1.-TFNSQ2))/DET**2
      CHCHC=((D21*EZ-D11*EX)**2*(1.-TFNSQ2))/(DET**2*G2D**2*TFNSQ2)
      CECH =((D12*EX-D22*EZ)*(D21*EZ-D11*EX)*(1.-TFNSQ2))/
     1      (DET**2*TFN22)
      CECHC=CMPLX(0.0,CECH)
      CECCH=-CECHC
      CECECP=((D12*EX-D22*EZ)**2)/DET**2
      CHCHCP=((D21*EZ-D11*EX)**2)/(DET**2*G2D**2*TFNSQ2)
      CECHP =((D12*EX-D22*EZ)*(D21*EZ-D11*EX))/
     1      (DET**2*TFN22)
      CECHCP=CMPLX(0.0,CECHP)
      CECCHP=-CECHCP
C
C
      IF(G3DSQ) 28,29,29
C
C   G2DSQ IS GREATER THAN ZERO AND  G3DSQ IS LESS THAN ZERO
 28   BEBE1=((WEPS3D*G3D**2*KC2DSQ)**2*CECEC)/((WEPS2D*G2D*KC3DSQ*
     1      TFN23)**2)
      BEBE2=(2.*WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC3DSQ-KC2DSQ)*
     1      CECH)/((WEPS2D*G2D*KC3DSQ)**2*TFN23)
      BEBE3=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CHCHC)/
     1      ((WEPS2D*G2D*KC3DSQ)**2)
      BEBEC=BEBE1+BEBE2+BEBE3
      BHBH1=((ALFD*BETAD*(KC2DSQ-KC3DSQ))**2*CECEC)/
     1      ((WMUD*G2D*KC3DSQ)**2)
      BHBH2=(2.*ALFD*BETAD*(KC2DSQ-KC3DSQ)*KC2DSQ*TFN23*CECH)/
     1      (WMUD*G2D**2*KC3DSQ**2)
      BHBH3=((KC2DSQ*TFN23)**2*CHCHC)/((G2D*KC3DSQ)**2)
      BHBHC=BHBH1+BHBH2+BHBH3
      BEBH1=(WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECEC)/
     1      (WEPS2D*G2D**2*KC3DSQ**2*TFN23*WMUD)
      BEBH2=(WEPS3D*G3D**2*KC2DSQ**2*CECH)/(WEPS2D*G2D**2*KC3DSQ**2)
      BEBH3=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CECH)/
     1      (WEPS2D*G2D**2*KC3DSQ**2*WMUD)
      BEBH4=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*KC2DSQ*TFN23*CHCHC)/
     1      (WEPS2D*G2D**2*KC3DSQ**2)
      BEBHC=CMPLX(0.0,-BEBH1)+CMPLX(0.0,-BEBH2)+
     1      CMPLX(0.0,BEBH3)+CMPLX(0.0,-BEBH4)
      BECBH=-BEBHC
```

```
      BECH1=(WEPS3D*G3D**2*KC2DSQ*CECH)/(WEPS2D*G2D*KC3DSQ*TFN23)
      BECH2=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CHCHC)/(WEPS2D*G2D*KC3DSQ)
      BECHC=CMPLX(0.0,BECH1)+CMPLX(0.0,BECH2)
      BECCH=-BECHC
      BHCE1=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECEC)/(WMUD*G2D*KC3DSQ)
      BHCE2=(KC2DSQ*TFN23*CECH)/(G2D*KC3DSQ)
      BHCEC=CMPLX(0.0,BHCE1)+CMPLX(0.0,BHCE2)
      BHCCE=-BHCEC
      BECE1=(WEPS3D*G3D**2*KC2DSQ*CECEC)/(WEPS2D*G2D*KC3DSQ*TFN23)
      BECE2=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CECH)/(WEPS2D*G2D*KC3DSQ)
      BECEC=CMPLX(BECE1,0.0)+CMPLX(BECE2,0.0)
      BECCE=BECEC
      BHCH1=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECH)/(WMUD*G2D*KC3DSQ)
      BHCH2=(KC2DSQ*TFN23*CHCHC)/(G2D*KC3DSQ)
      BHCHC=CMPLX(-BHCH1,0.0)-CMPLX(BHCH2,0.0)
      BHCCH=BHCHC
C
C
      BEBE1P=((WEPS3D*G3D**2*KC2DSQ)**2*CECECP)/((WEPS2D*G2D*
     1       KC3DSQ*TFN23)**2)
      BEBE2P=(2.*WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC3DSQ-KC2DSQ)*
     1       CECHP)/((WEPS2D*G2D*KC3DSQ)**2*TFN23)
      BEBE3P=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CHCHCP)/
     1       ((WEPS2D*G2D*KC3DSQ)**2)
      BEBECP=BEBE1P+BEBE2P+BEBE3P
      BHBH1P=((ALFD*BETAD*(KC2DSQ-KC3DSQ))**2*CECECP)/
     1       ((WMUD*G2D*KC3DSQ)**2)
      BHBH2P=(2.*ALFD*BETAD*(KC2DSQ-KC3DSQ)*KC2DSQ*TFN23*CECHP)/
     1       (WMUD*G2D**2*KC3DSQ**2)
      BHBH3P=((KC2DSQ*TFN23)**2*CHCHCP)/((G2D*KC3DSQ)**2)
      BHBHCP=BHBH1P+BHBH2P+BHBH3P
      BEBH1P=(WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC2DSQ-KC3DSQ)*
     1       CECECP)/(WEPS2D*G2D**2*KC3DSQ**2*TFN23*WMUD)
      BEBH2P=(WEPS3D*G3D**2*KC2DSQ**2*CECHP)/(WEPS2D*G2D**2*
     1       KC3DSQ**2)
      BEBH3P=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CECHP)/
     1       (WEPS2D*G2D**2*KC3DSQ**2*WMUD)
      BEBH4P=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*KC2DSQ*TFN23*CHCHCP)/
     1       (WEPS2D*G2D**2*KC3DSQ**2)
      BEBHCP=CMPLX(0.0,-BEBH1P)+CMPLX(0.0,-BEBH2P)+
     1       CMPLX(0.0,BEBH3P)+CMPLX(0.0,-BEBH4P)
      BECBHP=-BEBHCP
      BECH1P=(WEPS3D*G3D**2*KC2DSQ*CECHP)/(WEPS2D*G2D*KC3DSQ*TFN23)
      BECH2P=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CHCHCP)/(WEPS2D*G2D*KC3DSQ)
      BECHCP=CMPLX(0.0,BECH1P)+CMPLX(0.0,BECH2P)
      BECCHP=-BECHCP
      BHCE1P=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECECP)/(WMUD*G2D*KC3DSQ)
      BHCE2P=(KC2DSQ*TFN23*CECHP)/(G2D*KC3DSQ)
```

```
      BHCECP=CMPLX(0.0,BHCE1P)+CMPLX(0.0,BHCE2P)
      BHCCEP=-BHCECP
      BECE1P=(WEPS3D*G3D**2*KC2DSQ*CECECP)/(WEPS2D*G2D*KC3DSQ*TFN23)
      BECE2P=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CECHP)/(WEPS2D*G2D*KC3DSQ)
      BECECP=CMPLX(BECE1P,0.0)+CMPLX(BECE2P,0.0)
      BECCEP=BECECP
      BHCH1P=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECHP)/(WMUD*G2D*KC3DSQ)
      BHCH2P=(KC2DSQ*TFN23*CHCHCP)/(G2D*KC3DSQ)
      BHCHCP=CMPLX(-BHCH1P,0.0)-CMPLX(BHCH2P,0.0)
      BHCCHP=BHCHCP
C
C
C     POWER IN REGION 2
      P2A=BETAD*WEPS2D*ALFD**2
      P2B=BETAD*WMUD*ALFD**2
      P2C=BETAD*WEPS2D*G2D**2
      P2D=BETAD*WMUD*G2D**2
      P2E=BETAD**2*ALFD*G2D
      P2F=K2DSQ*ALFD*G2D
      P2G=BETAD*WEPS2D*(ALFD**2+G2D**2)
      P2H=BETAD*WMUD*(ALFD**2+G2D**2)
      P2I=2.*TFN22/G2D**2
      P2J=2.*TFNSQ2/G2D
      P21=(P2A*BEBEC+P2B*BHBHC+P2C*CECEC+P2D*CHCHC+C*P2E*(BECHC+
     1    BHCCE)-C*P2F*(BECCH+BHCEC))*(-2.)+
     1    (P2A*CECEC+P2B*CHCHC+P2C*BEBEC+P2D*BHBHC+C*P2E*(BHCCE+
     1    BECHC)-C*P2F*(BHCEC+BECCH))*2.
      P22=(P2A*BEBECP+P2B*BHBHCP+P2C*CECECP+P2D*CHCHCP+C*P2E*(BECHCP+
     1    BHCCEP)-C*P2F*(BECCHP+BHCECP))*P2I+
     1    (P2A*CECECP+P2B*CHCHCP+P2C*BEBECP+P2D*BHBHCP+C*P2E*(BHCCEP+
     1    BECHCP)-C*P2F*(BHCECP+BECCHP))*P2I+
     1    (BEBHCP+CECHCP)-C*2.*P2F*(CECCHP+BECBHP))*P2J
      P2 =P21+P22
C
C
      DEDEC=(KC2DSQ**2*(1.+TFNSQ3)*CECEC)/(KC3DSQ**2*TFNSQ3)
      DHDHC=(KC2DSQ**2*(1.+TFNSQ3)*CHCHC)/(KC3DSQ**2)
      DEDH =(KC2DSQ**2*(1.+TFNSQ3)*CECH*G3D)/(KC3DSQ**2*TFN23)
      DEDHC=CMPLX(DEDH,0.0)
      DECDH=DEDHC
C
C
C     POWER IN REGION 3
      P31=((BETAD*WEPS3D*ALFD**2*DEDEC/G3D)+(BETAD*WMUD*G3D*DHDHC)+
     1   (BETAD**2*ALFD*DEDHC)+(K3DSQ*ALFD*DECDH))*(2.*G3D*H2OVD-
     1   SIN(2.*G3D*H2OVD))
      P32=((BETAD*WMUD*ALFD**2*DHDHC/G3D)+(BETAD*WEPS3D*G3D*DEDEC)-
     1   (BETAD**2*ALFD*DEDHC)-(K3DSQ*ALFD*DECDH))*(2.*G3D*H2OVD+
```

```
     1      SIN(2.*G3D*H2OVD))
          P3=P31+P32
          GO TO 30
C
C
C    G2DSQ IS GREATER THAN ZERO AND  G3DSQ IS GREATER THAN ZERO
  29      BEBE1=((WEPS3D*G3D**2*KC2DSQ)**2*CECEC)/((WEPS2D*G2D*KC3DSQ*
     1         TFN23)**2)
          BEBE2=(2.*WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC3DSQ-KC2DSQ)*
     1         CECH)/((WEPS2D*G2D*KC3DSQ)**2*TFN23)
          BEBE3=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CHCHC)/
     1         ((WEPS2D*G2D*KC3DSQ)**2)
          BEBEC=BEBE1+BEBE2+BEBE3
          BHBH1=((ALFD*BETAD*(KC2DSQ-KC3DSQ))**2*CECEC)/
     1         ((WMUD*G2D*KC3DSQ)**2)
          BHBH2=(2.*ALFD*BETAD*(KC2DSQ-KC3DSQ)*KC2DSQ*TFN23*CECH)/
     1         (WMUD*G2D**2*KC3DSQ**2)
          BHBH3=((KC2DSQ*TFN23)**2*CHCHC)/((G2D*KC3DSQ)**2)
          BHBHC=BHBH1-BHBH2+BHBH3
          BEBH1=(WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECEC)/
     1         (WEPS2D*G2D**2*KC3DSQ**2*TFN23*WMUD)
          BEBH2=(WEPS3D*G3D**2*KC2DSQ**2*CECH)/(WEPS2D*G2D**2*KC3DSQ**2)
          BEBH3=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CECH)/
     1         (WEPS2D*G2D**2*KC3DSQ**2*WMUD)
          BEBH4=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*KC2DSQ*TFN23*CHCHC)/
     1         (WEPS2D*G2D**2*KC3DSQ**2)
          BEBHC=CMPLX(0.0,-BEBH1)+CMPLX(0.0,BEBH2)+
     1         CMPLX(0.0,BEBH3)+CMPLX(0.0,BEBH4)
          BECBH=-BEBHC
          BECH1=(WEPS3D*G3D**2*KC2DSQ*CECH)/(WEPS2D*G2D*KC3DSQ*TFN23)
          BECH2=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CHCHC)/(WEPS2D*G2D*KC3DSQ)
          BECHC=CMPLX(0.0,BECH1)+CMPLX(0.0,BECH2)
          BECCH=-BECHC
          BHCE1=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECEC)/(WMUD*G2D*KC3DSQ)
          BHCE2=(KC2DSQ*TFN23*CECH)/(G2D*KC3DSQ)
          BHCEC=CMPLX(0.0,BHCE1)-CMPLX(0.0,BHCE2)
          BHCCE=-BHCEC
          BECE1=(WEPS3D*G3D**2*KC2DSQ*CECEC)/(WEPS2D*G2D*KC3DSQ*TFN23)
          BECE2=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CECH)/(WEPS2D*G2D*KC3DSQ)
          BECEC=CMPLX(BECE1,0.0)+CMPLX(BECE2,0.0)
          BECCE=BECEC
          BHCH1=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECH)/(WMUD*G2D*KC3DSQ)
          BHCH2=(KC2DSQ*TFN23*CHCHC)/(G2D*KC3DSQ)
          BHCHC=CMPLX(-BHCH1,0.0)+CMPLX(BHCH2,0.0)
          BHCCH=BHCHC
C
C
          BEBE1P=((WEPS3D*G3D**2*KC2DSQ)**2*CECECP)/((WEPS2D*G2D*KC3DSQ*
```

```
     1      TFN23)**2)
       BEBE2P=(2.*WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC3DSQ-KC2DSQ)*
     1      CECHP)/((WEPS2D*G2D*KC3DSQ)**2*TFN23)
       BEBE3P=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CHCHCP)/
     1      ((WEPS2D*G2D*KC3DSQ)**2)
       BEBECP=BEBE1P+BEBE2P+BEBE3P
       BHBH1P=((ALFD*BETAD*(KC2DSQ-KC3DSQ))**2*CECECP)/
     1      ((WMUD*G2D*KC3DSQ)**2)
       BHBH2P=(2.*ALFD*BETAD*(KC2DSQ-KC3DSQ)*KC2DSQ*TFN23*CECHP)/
     1      (WMUD*G2D**2*KC3DSQ**2)
       BHBH3P=((KC2DSQ*TFN23)**2*CHCHCP)/((G2D*KC3DSQ)**2)
       BHBHCP=BHBH1P-BHBH2P+BHBH3P
       BEBH1P=(WEPS3D*G3D**2*KC2DSQ*ALFD*BETAD*(KC2DSQ-KC3DSQ)*
     1       CECECP)/(WEPS2D*G2D**2*KC3DSQ**2*TFN23*WMUD)
       BEBH2P=(WEPS3D*G3D**2*KC2DSQ**2*CECHP)/(WEPS2D*G2D**2*
     1       KC3DSQ**2)
       BEBH3P=((ALFD*BETAD*(KC3DSQ-KC2DSQ))**2*CECHP)/
     1      (WEPS2D*G2D**2*KC3DSQ**2*WMUD)
       BEBH4P=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*KC2DSQ*TFN23*CHCHCP)/
     1      (WEPS2D*G2D**2*KC3DSQ**2)
       BEBHCP=CMPLX(0.0,-BEBH1P)+CMPLX(0.0,BEBH2P)+
     1      CMPLX(0.0,BEBH3P)+CMPLX(0.0,BEBH4P)
       BECBHP=-BEBHCP
       BECH1P=(WEPS3D*G3D**2*KC2DSQ*CECHP)/(WEPS2D*G2D*KC3DSQ*TFN23)
       BECH2P=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CHCHCP)/(WEPS2D*G2D*KC3DSQ)
       BECHCP=CMPLX(0.0,BECH1P)+CMPLX(0.0,BECH2P)
       BECCHP=-BECHCP
       BHCE1P=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECECP)/(WMUD*G2D*KC3DSQ)
       BHCE2P=(KC2DSQ*TFN23*CECHP)/(G2D*KC3DSQ)
       BHCECP=CMPLX(0.0,BHCE1P)-CMPLX(0.0,BHCE2P)
       BHCCEP=-BHCECP
       BECE1P=(WEPS3D*G3D**2*KC2DSQ*CECECP)/(WEPS2D*G2D*KC3DSQ*TFN23)
       BECE2P=(ALFD*BETAD*(KC3DSQ-KC2DSQ)*CECHP)/(WEPS2D*G2D*KC3DSQ)
       BECECP=CMPLX(BECE1P,0.0)+CMPLX(BECE2P,0.0)
       BECCEP=BECECP
       BHCH1P=(ALFD*BETAD*(KC2DSQ-KC3DSQ)*CECHP)/(WMUD*G2D*KC3DSQ)
       BHCH2P=(KC2DSQ*TFN23*CHCHCP)/(G2D*KC3DSQ)
       BHCHCP=CMPLX(-BHCH1P,0.0)+CMPLX(BHCH2P,0.0)
       BHCCHP=BHCHCP
C
C
C    POWER IN REGION 2
       P2A=BETAD*WEPS2D*ALFD**2
       P2B=BETAD*WMUD*ALFD**2
       P2C=BETAD*WEPS2D*G2D**2
       P2D=BETAD*WMUD*G2D**2
       P2E=BETAD**2*ALFD*G2D
       P2F=K2DSQ*ALFD*G2D
```

```
      P2G=BETAD*WEPS2D*(ALFD**2+G2D**2)
      P2H=BETAD*WMUD*(ALFD**2+G2D**2)
      P2I=2.*TFN22/G2D**2
      P2J=2.*TFNSQ2/G2D
      P21=(P2A*BEBEC+P2B*BHBHC+P2C*CECEC+P2D*CHCHC+C*P2E*(BECHC+
     1    BHCCE)-C*P2F*(BECCH+BHCEC))*(-2.)+
     1    (P2A*CECEC+P2B*CHCHC+P2C*BEBEC+P2D*BHBHC+C*P2E*(BHCCE+
     1    BECHC)-C*P2F*(BHCEC+BECCH))*2.
      P22=(P2A*BEBECP+P2B*BHBHCP+P2C*CECECP+P2D*CHCHCP+C*P2E*(BECHCP+
     1    BHCCEP)-C*P2F*(BECCHP+BHCECP))*P2I+
     1    (P2A*CECECP+P2B*CHCHCP+P2C*BEBECP+P2D*BHBHCP+C*P2E*(BHCCEP+
     1    BECHCP)-C*P2F*(BHCECP+BECCHP))*P2I+
     1    (P2G*(BECECP+BECCEP)+P2H*(BHCHCP+BHCCHP)+C*2.*P2E*
     1    (BEBHCP+CECHCP)-C*2.*P2F*(CECCHP+BECBHP))*P2J
      P2 =P21+P22
C
C
      DEDEC=(KC2DSQ**2*(1.-TFNSQ3)*CECEC)/(KC3DSQ**2*TFNSQ3)
      DHDHC=(KC2DSQ**2*(1.-TFNSQ3)*CHCHC)/(KC3DSQ**2)
      DEDH =(KC2DSQ**2*(1.-TFNSQ3)*CECH*G3D)/(KC3DSQ**2*TFN23)
      DEDHC=CMPLX(0.0,DEDH)
      DECDH=-DEDHC
      DEDECP=(KC2DSQ**2*CECEC)/(KC3DSQ**2*TFNSQ3)
      DHDHCP=(KC2DSQ**2*CHCHC)/(KC3DSQ**2)
      DEDHP =(KC2DSQ**2*CECH*G3D)/(KC3DSQ**2*TFN23)
      DEDHCP=CMPLX(0.0,DEDHP)
      DECDHP=-DEDHCP
C
C
C   POWER IN REGION 3
      P3A=BETAD*WEPS3D*ALFD**2
      P3B=BETAD*WMUD*G3D**2
      P3C=BETAD**2*ALFD*G3D
      P3D=K3DSQ*ALFD*G3D
      P3E=BETAD*WMUD*ALFD**2
      P3F=BETAD*WEPS3D*G3D**2
      P3G=2.*TFN23/G3D**2
      P31=(P3A*DEDEC+P3B*DHDHC+C*P3C*DEDHC-C*P3D*DECDH)*(-2.*H2OVD)+
     1    (P3E*DHDHC+P3F*DEDEC+C*P3C*DEDHC-C*P3D*DECDH)*(2.*H2OVD)
      P32=(P3A*DEDECP+P3B*DHDHCP+C*P3C*DEDHCP-C*P3D*DECDHP)*P3G+
     1    (P3E*DHDHCP+P3F*DEDECP+C*P3C*DEDHCP-C*P3D*DECDHP)*P3G
      P3 =P31+P32
C
C
 30   PWR=PWR+P1+P2+P3
      PWR1=PWR1+REAL(P1)
      PWR2=PWR2+REAL(P2)
      PWR3=PWR3+REAL(P3)
```

```
           IF(ALFD.EQ.0.) PWR=PWR/2.
           IF(ALFD.EQ.0.) PWR1=PWR1/2.
           IF(ALFD.EQ.0.) PWR2=PWR2/2.
           IF(ALFD.EQ.0.) PWR2=PWR2/2.
   14 CONTINUE
          IMP=0.5/(0.25*REAL(PWR)/BOVD)
          WRITE(01,500) FREQ,IMP
C         WRITE(01,500) FREQ,LPOVL,IMP
   11       CONTINUE
   12 CONTINUE
   13 STOP
C     FORMAT FOR  FREQ, D/L, LP/L, IMPEDANCE
  500 FORMAT(F5.2,F8.3)
C 500 FORMAT(3F10.4)
  610 FORMAT('+',29X,'   MSG2    '/)
C MSG2: SEARCH FOR ZERO OF IPROD1 FAILED TO CONVERGE
      END
C SUBR*******************************************************************
      SUBROUTINE IPROD1(LPOVL,SUM1)
C***********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE INNER PRODUCT OF   *
C THE G11 DYADIC GREEN'S FUNCTION AND THE SQUARE OF THE FOURIER TRANS*
C -FORM OF THE X-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD          *
C*****VARIABLE DEFINITIONS**********************************************
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE   *
C         X-DIRECTED ELECTRIC FIELD                                   *
C G11 = THE G11 DYADIC GREEN'S FUNCTION                               *
C MMAX = THE LIMIT OF SUMMATION OVER ALFD                             *
C SUM1 = THE RETURNED VALUE OF THE INNER PRODUCT                      *
C*****VARIABLE DECLARATION**********************************************
      REAL LPOVL
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      COMMON/C3/DOVL,WOVB
      COMMON/C5/XCONST
      BETAD = C2PI*DOVL/LPOVL
      BOVW = 1./WOVB
C   SPECIFY LIMIT FOR SUMMATION ON N
      IF(WOVB.EQ.1.) MMAX=1
      IF(WOVB.LT.1.) MMAX=INT((XCONST*BOVW)/C2PI)
      IF(MMAX.GT.40) MMAX=40
      SUM1  = 0.0
      DO 11 M=1,MMAX
         N=M-1
         ALFD=N*C2PI/BOVD
         CALL GRN11(ALFD,BETAD,DOVL,G11)
         CALL EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
         SUM1=SUM1+2.*G11*EXXSQ
```

```
      IF(N.EQ.0) SUM1=G11
   11 CONTINUE
      RETURN
      END
C SUBR*****************************************************************
      SUBROUTINE GRN11(ALFD,BETAD,DOVL,G11)
C*********************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE G11 DYADIC GREEN'S*
C FUNCTION                                                           *
C*****VARIABLE DEFINITIONS********************************************
C G11 = THE G11 DYADIC GREEN'S FUNCTION, RETURNED VALUE              *
C DET                                                                *
C D11                                                                *
C D12                                                                *
C D21                                                                *
C D22                                                                *
C G1DSQ                                                              *
C G2DSQ                                                              *
C G3DSQ                                                              *
C G11                                                                *
C G11A                                                               *
C G11B                                                               *
C G11C                                                               *
C G11D                                                               *
C G11E                                                               *
C KC1DSQ                                                             *
C KC2DSQ                                                             *
C KC3DSQ                                                             *
C TFN                                                                *
C TFN1                                                               *
C TFN2                                                               *
C TFN3                                                               *
C WEPS1D                                                             *
C WEPS2D                                                             *
C WEPS3D                                                             *
C WMUD                                                               *
C*****VARIABLE DECLARATION********************************************
      COMMON/C1/EPSR1,EPSR2,EPSR3,H1OVD,H2OVD,BOVD
      COMMON/C2/C2PI,C2PISQ,PI
      REAL KC1DSQ,KC2DSQ,KC3DSQ,LPROVL,LOVLPR
C   CALCULATE VARIABLES DEPENDENT ON FREQUENCY
      WMUD=60.*C2PISQ*DOVL
      WEPS1D=EPSR1*DOVL/60.
      WEPS2D=EPSR2*DOVL/60.
      WEPS3D=EPSR3*DOVL/60.
C   CALCULATE VARIABLES DEPENDENT ON FRRQUENCY AND BETAD
      BETDSQ=BETAD**2
      KC1DSQ=C2PISQ*EPSR1*DOVL**2-BETDSQ
```

```
      KC2DSQ=C2PISQ*EPSR2*DOVL**2-BETDSQ
      KC3DSQ=C2PISQ*EPSR3*DOVL**2-BETDSQ
C   CALCULATE VARIABLES DEPENDENT ON FREQUENCY, BETAD AND ALFD
      ALFDSQ=ALFD**2
      G1DSQ=ALFDSQ-KC1DSQ
      G2DSQ=ALFDSQ-KC2DSQ
      G3DSQ=ALFDSQ-KC3DSQ
      CALL TFN(G1DSQ,H1OVD,TFN1)
      CALL TFN(G2DSQ,1.,TFN2)
      CALL TFN(G3DSQ,H2OVD,TFN3)
      D11=-KC2DSQ*(1.+(WEPS3D*G3DSQ*KC2DSQ*TFN2)/(WEPS2D*G2DSQ*KC3DSQ*
     1TFN3))
      D12=((ALFD*BETAD)/(WEPS2D*G2DSQ))*((KC2DSQ/KC1DSQ)-1.)*KC2DSQ
      D21=-ALFD*BETAD*((KC2DSQ/KC1DSQ)+((WEPS3D*KC2DSQ*G3DSQ*TFN2)/
     1(WEPS2D*KC3DSQ*G2DSQ*TFN3)))
      D22=WMUD*(1.+((KC2DSQ*TFN3)/(KC3DSQ*TFN2))+((ALFDSQ*BETDSQ)/(G2DSQ
     1*WMUD*WEPS2D))*((KC2DSQ/KC3DSQ)-1.))
      DET=D11*D22-D21*D12
C   CALCULATE G11
      G11A=-KC1DSQ/(WMUD*TFN1)
      G11B=-(KC2DSQ*D12*ALFD*BETAD*KC2DSQ*TFN2)/(DET*WMUD*G2DSQ*KC3DSQ)
      G11C=(KC2DSQ*D12*ALFD*BETAD*TFN2)/(DET*WMUD*G2DSQ)
      G11D=-(KC2DSQ*D11*KC2DSQ*TFN3)/(DET*KC3DSQ*G2DSQ)
      G11E=-(KC2DSQ*D11)/(DET*TFN2)
      G11=G11A+G11B+G11C+G11D+G11E
      RETURN
      END
C SUBR*****************************************************************
      SUBROUTINE TFN(GIDSQ,HIOVD,TFNI)
C*******************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE TANGENT AND THE    *
C HYPERBOLIC TANGENT FUNCTIONS FOR THE GRN11 SUBROUTINE.              *
C*****VARIABLE DEFINITIONS*******************************************
C GID                                                                 *
C GIDSQ                                                               *
C HIOVD                                                               *
C TFNI = THE RETURNED VALUE                                           *
C*****VARIABLE DECLARATION*******************************************
      GID=SQRT(ABS(GIDSQ))
      ARG=GIDSQ*HIOVD**2
      IF(ARG.LE.0.) GO TO 1
      IF((ARG.GT.0.).AND.(ARG.LT.100.)) GO TO 2
      IF(ARG.GE.100.) GO TO 3
    1 TFNI=-GID*TAN(GID*HIOVD)
      GO TO 4
    2 TFNI=GID*TANH(GID*HIOVD)
      GO TO 4
    3 TFNI=GID
```

```
    4 RETURN
      END
C SUBR*************************************************************
      SUBROUTINE TFNS(GIDSQ,HIOVD,TFN2I)
C*****************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE TANGENT AND THE   *
C HYPERBOLIC TANGENT FUNCTIONS FOR THE GRN11 SUBROUTINE.             *
C*****VARIABLE DEFINITIONS******************************************
C GID                                                                *
C GIDSQ                                                              *
C HIOVD                                                              *
C TFNI = THE RETURNED VALUE                                          *
C*****VARIABLE DECLARATION******************************************
      GID=SQRT(ABS(GIDSQ))
      ARG=GIDSQ*HIOVD**2
      IF(ARG.LE.0.) GO TO 1
      IF(ARG.GT.0.) GO TO 2
    1 TFN2I=GID*TAN(GID*HIOVD)
      GO TO 3
    2 TFN2I=GID*TANH(GID*HIOVD)
    3 RETURN
      END
C SUBR*************************************************************
      SUBROUTINE TFNSQ (GIDSQ,HIOVD,TFNSQI)
C*****************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE TANGENT AND THE   *
C HYPERBOLIC TANGENT FUNCTIONS FOR GENERAL PURPOSE.                  *
C*****VARIABLE DEFINITIONS******************************************
C GID                                                                *
C GIDSQ                                                              *
C HIOVD                                                              *
C TFNSQI = THE RETURNED VALUE                                        *
C*****VARIABLE DECLARATION******************************************
      GID=SQRT(ABS(GIDSQ))
      ARG=GIDSQ*HIOVD**2
      IF(ARG.LE.0.) GO TO 1
      IF(ARG.GT.0.) GO TO 2
    1 TFNSQI=(TAN(GID*HIOVD))**2
      GO TO 3
    2 TFNSQI=(TANH(GID*HIOVD))**2
    3 RETURN
      END
C SUBR*************************************************************
      SUBROUTINE EXSQ1(ALFD,WOVB,BOVD,EXXSQ)
C*****************************************************************
C     THIS SUBROUTINE IS DESIGNED TO CALCULATE THE FOURIER TRANSFORM *
C OF THE X-COMPONENT OF THE X-DIRECTED ELECTRIC FIELD AND SQUARE IT. *
C*****VARIABLE DEFINITIONS******************************************
```

```
C EXXSQ = SQUARE OF THE FOURIER TRANSFORM OF THE X-COMPONENT OF THE  *
C           X-DIRECTED ELECTRIC FIELD, THE RETURNED VALUE            *
C*****VARIABLE DECLARATION*********************************************
      COMMON/C2/C2PI,C2PISQ,PI
      WOVD=WOVB*BOVD
C     CALCULATE EXXSQ, THE X TRANSFORM OF THE E FIELD BETWEEN THE FINS
      IF(ALFD.EQ.0.) EXXSQ=1.
      IF(ALFD.GT.0.) EXXSQ=(SIN(.5*ALFD*WOVD)/(.5*ALFD*WOVD))**2
      RETURN
      END
```

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c.1
A computation of fin-line impedence.